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An Approximation for Computing Reduction in Bandwidth Requirements using Intelligent Multiplexers

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Submitted in partial fulfillment of the
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ABSTRACT

This paper stochastically models a single-node telecommunications system both with and without the use of intelligent multiplexing. Intelligent multiplexers take advantage of the idle periods or silences that occur during the course of speech transmissions to merge (or multiplex) packetized talkspurts from more than one source onto a single channel. This allows for a more efficient use of available bandwidth, thereby reducing the amount of bandwidth required to carry a particular traffic load. Digitizing speech into packets of equal size also allows for compression, further reducing bandwidth needs. By comparing the models for systems both with and without multiplexing, we are able to determine the reduction in bandwidth which may be expected for a particular grade of service (measured by blocking probabilities). A bivariate continuous time Markov chain model for a multiplexer is presented. An approximation is introduced to calculate limiting blocking probabilities much more quickly and for larger systems than is possible by solving a set of linear equations for the bivariate model. The accuracy of the approximation is explored through comparison with the bivariate model; the approximation provides a somewhat conservative estimate of blocking, but is close enough to be used as a tool for the range of relevant values. The approximation is then used to compare blocking probabilities for three different levels of speech activity. Results are shown in tabular form.

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The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

I. INTRODUCTION

The field of telecommunications has been advancing at a tremendous rate in recent years, assisted by the decreasing costs and increasing capability of microprocessors, as well as by deregulation of the industry. New products and capabilities are coming online at an astounding rate. It has become commonplace to transport data between computers with the use of modems along standard telephone lines at ever-increasing baud rates. More companies every day are opting to use video-conferencing as a replacement for time-consuming travel to business meetings. Fax machines are now priced for use in the home as well as in the office. Cellular phones for use in automobiles and airplanes make "getting away from it all" more difficult than ever. The latest sensation to hit the consumer market is a telephone with video screen to view the person on the other end of the phone line (if they have the same device, of course), also priced for home use. There seems to be no limit to the potential market for increasingly sophisticated (i.e. bandwidth intensive) telecommunications products.

In order to provide economical transmission of high bandwidth data, such as fax and video, it has become increasingly important to find inexpensive ways to increase bandwidth and to conserve the bandwidth available. A variety of technical innovations, such as fiber optic networks, data compression techniques, and multiplexers, have been developed to do just that.

A. WHAT IS MULTIPLEXING?

Multiplexing techniques are designed to reduce bandwidth needs, thereby reducing costs, by sharing bandwidth among network users. Intelligent multiplexers accomplish this by sending the packetized information from a large number of channels onto a single wideband channel, without transmitting any of the silent periods. This achieves very high utilization rates along the single channel.

Intelligent multiplexers take advantage of the idle times that occur during the course of any telecommunications transmission to make more efficient use of available bandwidth. Speech conversations, for instance, are silent about 60% of the time; when one person is speaking, the other is normally silent and listens. Also, there are pauses between words and sentences. Data traffic often averages only 5-15% efficiency, tending to be bursty, occurring for a short time, then subsiding to occur some undetermined time later. These bursts of data traffic also have high bandwidth requirements.

There are two basic types of intelligent multiplexer on the market. The older of the two is referred to as statistical time-division multiplexing (STDM) or statistical packet multiplexing (SPM); the newer is called fast packet multiplexing (FPM). They are both microprocessor-based, meaning both higher efficiency and higher cost when compared with frequency division multiplexing and time division multiplexing. These newer technologies will consequently only begin to replace what is already in use as microprocessor price/performance ratios improve enough to justify the efficiency gains.

B. STANDARDS

Due to the current lack of standards for intelligent multiplexing equipment, manufacturers have each designed their own intermachine communication systems, and no two systems are compatible. This creates problems when users of private networks want to tap into another private or public network. It can also make direct comparisons among various vendor products difficult for the potential buyer.

Standards bodies, such as the American National Standards Institute (ANSI), the International Telegraph and Telephone Consultative Committee (CCITT), and the Institute of Electrical and Electronic Engineers (IEEE), are working on standards for equipment which will likely supersede current multiplexer technology. Standards seem to be evolving in the direction of transmitting all information (speech, data, video, etc.) in the form of packets or "cells."

C. DEFENSE COMMUNICATIONS AGENCY INTEREST

The Defense Communications Agency (DCA) is extremely interested in exploring the capabilities of these new and emerging technologies in order to plan ahead for changes to MILDEP networks. Studies are ongoing to assess the various intelligent or "smart" multiplexer products on the market and to determine criteria on which to base future purchasing decisions [Ref. 1: pp. 1-23].

In the Advanced Design Group, headed by Dr. Martin Fischer, the inclusion of intelligent multiplexers (smart mux or smux) will affect the network topology design tools currently being developed. The key question for them, regarding the smart mux, is how much of a reduction in bandwidth can be obtained by the use of intelligent multiplexers while maintaining current network performance levels. Their data base contains bandwidth costs based on AT&T tariffs as well as the cost data for several different brands of multiplexer. They also know how many channels are required to carry a particular traffic load, expressed in Erlangs, without the use of a smart multiplexer. A simple way to calculate the reduction in channels needed when multiplexers are added to the network would allow them to do comparative cost analyses.

D. PURPOSE OF THIS STUDY

The purpose of this study is to find a simple, yet relatively accurate, way to determine the reduction in bandwidth which will result from adding intelligent multiplexers to a voice network. It will involve stochastically modeling a single node of a communications network, both with and without a multiplexer. Approximations to the more complicated stochastic model are then studied.

In the next section we provided a summary of the technology. In Section III we review some of the relevant literature. Section IV presents a description of the models studied, while Section V covers the approximation techniques used to compute limiting probabilities for those models. In Section VI we describe the programs used to perform the calculations and the validation techniques for the computer code. In Sections VII and VIII we discuss the numerical results and conclusions, respectively.

II. DESCRIPTION OF THE TECHNOLOGY

A. FREQUENCY DIVISION MULTIPLEXING

The oldest multiplexing technique is frequency division multiplexing (FDM). FDM divides the frequency spectrum of analog circuits into smaller narrowband segments. The narrowband implementation limits the data rates which can be used for remote networking [Ref. 2: p. 54].

B. TIME DIVISION MULTIPLEXING

Time division multiplexing (TDM), which began to replace FDM when remote network data rates increased above 2400 bits per second (bps), divides the communication link into a fixed number of time slots. Each slot is assigned to a specific channel. Transmission occurs in a regular sequence, cycling through the channels. Bandwidth allocation is fixed, and is based on the size of the time slot allocated to each channel. TDM is relatively inexpensive to implement and introduces very little delay. However, TDM is not very efficient in the use of bandwidth. If a channel is idle, that time slot is not available for use by any other channel. Also, the silent periods of a voice or data transmission go unused. For combined voice and data traffic, TDM averages only 10-25% efficiency. TDM is unable to momentarily increase bandwidth for high-speed data due to the fixed time slots and bit rates. Thus, TDM is not well-suited to transporting a dynamically varying combination of voice, fax, and LAN traffic [Ref. 2: p. 54].

C. STATISTICAL PACKET MULTIPLEXING

Statistical packet multiplexing solves both of the problems associated with TDM, that is, network efficiency and ability to dynamically allocate bandwidth, but has two drawbacks of its own. It introduces higher network delay and difficulty in predicting the amount of delay. Thus, SPM is not suited for time-sensitive information, such as voice and video traffic.

Instead of statically dividing the network bandwidth as in TDM, SPM dynamically allocates bandwidth to those channels passing data at the moment. Within the multiplexer (mux), SPM operates by gathering transmitted data from the active channel into a packet, appending identifying and control information, and passing the packet to the next multiplexer. The next mux checks for transmission errors (using the control information) and requests retransmission if errors are found. Any errors are corrected before the packet is sent on. The packetization of data also allows the originating multiplexer to easily perform various operations on the data, such as encryption and compression.

Due to the different advantages and disadvantages associated with both TDM and SPM, many networks in use today are hybrids that combine the two. TDM is used for time-sensitive information (voice, video, some synchronous data and LAN traffic) while SPM is used where higher network efficiency and dynamic bandwidth allocation are important (primarily asynchronous data, and some synchronous data and LAN traffic) [Ref. 2: p.55].

Descriptions of the first three multiplexing techniques may be found in references [Ref. 2: pp. 54-55, Ref. 3: pp. 112-113, and Ref. 4: pp. 165-188].

D. FAST PACKET MULTIPLEXING

Fast packet multiplexing (FPM) is a generic term for remote networking techniques that satisfy the following criteria [Ref. 2:p. 54]:

- the ability to transport a dynamically varying combination of voice, fax, video, synchronous data, asynchronous data, and LAN (local area network) traffic;
- high network efficiency, typically 90% or better;
- low network delay;
- predictable delivery of time-sensitive information.

Fast packet multiplexing is the most recent of four main multiplexing techniques designed for use in telecommunications networks. It is very similar to statistical packet multiplexing. As with previous multiplexing techniques, it is a way to reduce bandwidth needs by sharing bandwidth among network users, thereby reducing costs.

Unlike the other multiplexing techniques, it is designed to efficiently transmit a wide variety of time-sensitive information along the same network.

FPM has the following characteristics [Ref. 2:pp. 56-59]:

- it gathers each incoming channel's data into equal size cells (packets) for delivery over the network;
- it begins to forward cells of a message before all cells are completely received; i.e. cells pass through the FPM device rather than into and then out of the device;
- it can interrupt the delivery of one channel's message in favor of delivering a more time-sensitive (i.e. higher priority) channel's message (using cell boundaries to determine where interruptions may occur);
- the time it takes to transmit a cell is directly related to both the cell size and the bit rate of the network (outgoing) link; low rates and large cell size increase transmission time. The cell size is fixed by making it

proportional to the bit rate of the network link. Since cell sizes and bit rates of the links are fixed, service times for each cell are equal;

- it eliminates idle bandwidth from the incoming channels and transmits only active information, so more calls can be in progress than the number of physical channels available.

III. LITERATURE REVIEW

A. QUEUEING THEORY

1. The Erlang B (Loss) Formula

Voice communication systems using time-division multiplexing are often modeled stochastically as queueing models, using the Erlang loss system [Ref. 5:pp. 79-81]. Here, it is assumed that calls are initiated according to a Poisson process with rate λ, service times are exponentially distributed with mean length μ^{-1}, independent of each other and the arrival process; and if all servers (channels) are busy when a customer (caller) arrives, that customer cannot enter the system (gets a busy signal); that is, blocked customers are cleared (BCC). The ratio λ/μ is the offered load a, expressed in Erlangs. For a given number of channels c, the limiting probability of j busy channels is given by the truncated Poisson distribution:

$$\lim_{t\to\infty} P_j(t) = P_j = \frac{\dfrac{(\lambda/\mu)^j}{j!}}{\displaystyle\sum_{k=0}^{c}\frac{(\lambda/\mu)^k}{k!}} \quad (j = 0,1,\ldots c) \tag{1}$$

This formulation is also found in Ross [Ref. 6:p. 390].

The proportion of time that all c channels are busy is calculated by the Erlang B formula (or Erlang loss formula)

$$B(c,a) = \frac{a^c/c!}{\sum_{k=0}^{c} a^k/k!}, \tag{2}$$

where $a = \lambda/\mu$.

This formula is used to determine the number of channels c needed to achieve a particular blocking probability $B(c,a)$, given the offered load a in Erlangs. By plotting the Erlang loss formula $B(c,a)$ against increasing values of a, curves for fixed values of c are obtained [Ref. 5:pp. 316-317]. Tables of these values have also been created. The carried load a' is also easily calculated:

$$a' = a\,[1 - B(c,a)]. \tag{3}$$

This is part of the method currently in use at DCA to determine the number of channels required along any particular trunk in the network modeling process for a given load.

B. MULTIPLEXER MODELS

Numerous models for various types of multiplexer have been developed. Similar models are used to analyze both computer and communication networks. A data-handling computer network is modeled by Anick, Mitra, and Sondhi [Ref. 7:pp. 1871-1894] using differential equations to describe the equilibrium buffer distribution. The model is used to determine the appropriate buffer size for a particular number of sources and grade of service. It is also used to determine the maximum number of sources to be allowed in the system. Integrated voice-data multiplexers are modeled in references [Ref. 8:pp. 8-14, Ref. 9:pp. 1124-1132, Ref. 10:pp. 833-846, and Ref. 11:pp. 1003-1009]. The first reference [Ref. 8:pp. 8-14] uses a continous-time queueing model which models performance of a flow control scheme for a movable boundary voice-data multiplexer and develops a decision rule based on data queue length to cutoff the priority of voice. Reference [Ref. 9:pp. 1124-1132] compares two different voice-data multiplexer schemes, both of which use the movable boundary frame allocation scheme. The second scheme uses speech activity detectors (SAD's) so that the multiplexer also performs digital speech interpolation. This allows utilization of talker silences for transmission of additional voice and/or data. Performance measures include: probability of loss for voice calls, probability of speech clipping, speech packet rejection ratio, and expected message delay. The third reference [Ref. 10:pp. 833-846] uses the index of dispersion for intervals (IDI) as a measurement tool to characterize the complex arrival process resulting from superposition of separate voice streams. The paper also describes delays experienced by voice and data packets using a two-parameter approximation. The fourth reference [Ref. 11:pp. 1003-1009] models wideband packet technology integrating packetized voice and data using statistical multiplexing. It incorporates a flexible bandwidth allocation scheme with bit dropping; results using simulation show good voice quality, low delay and packet loss, efficient use of transmission bandwidth, and protection in overload. References [Ref. 12:pp. 847-855, Ref. 13:pp. 41-56, Ref. 14:pp. 703-712, and Ref. 15:pp. 718-728] all model packet voice multiplexers. Reference [Ref. 12:pp. 847-855] describes three models; a semi-Markov process, a continuous-time Markov chain, and a uniform arrival and service model; then compares numerical results of the queueing behavior of the three models to each other and to a discrete-event simulation and an M/D/1 analysis. All models assume multiple independent voice sources which form a queue for first-in-first-out (FIFO) service along a finite-capacity communications link. The second reference [Ref. 13:pp. 41-56] develops methodologies for evaluating the performance of variable bit rate voice under the following two conditions: (1) at a fixed load when instantaneous fluctuations occur due to talker activity/inactivity and (2) under variable load when variations occur due to call on/off. The authors use a Markov chain model in conjunction with a software package to emulate packetized voice and describe the probabilistic bit-dropping pattern under various loading and traffic conditions. The third reference [Ref. 14:pp. 703-712] uses simulation and analytic modeling (M/D/1/K) to examine performance of a packet voice multiplexer queue which employs bit dropping during periods of congestion. Results indicate that significant capacity and performance advantages are gained in the multiplexer as a result of dropping the least significant bits when the system is congested. The fourth reference [Ref. 15:pp. 718-728] also uses an M/D/1/K queueing model for measuring performance of a voice packet network which uses bit dropping.

For purposes of this paper we have chosen a model which allows no queue to develop (blocked customers are cleared). Rather, we focus on the proportion of time that blocking occurs. That is, we assume that voice calls are so time-sensitive that no waiting time can be tolerated, so they are dropped (denied transmission) to avoid congestion. This is not a completely accurate description of what occurs in the multiplexer, however, we hope that it provides an adequate, albeit conservative approach.

IV. MODEL DEVELOPMENT

A. THE ERLANG MODEL

The first step toward developing the multiplexer model is to enhance the Erlang model with the addition of talkspurts. This will be used as a basis for the multiplexer model and also as a comparison model by which to measure the relative performance increase once a multiplexer is added.

1. Variables

In what follows, the following variables were used:

$C(t)$ is used to represent the number of calls in progress at time t.

$A(t)$ is used to represent the number of talkspurts (active calls) at time t.

K is the maximum number of calls allowed (= the number of channels).

Lambda (λ) is the call initiation rate (in call initiations per second).

Mu (μ) is the call termination rate (in call terminations per second).

μ^{-1} is the mean time (in seconds) that a call is in progress.

Alpha (α) is the talkspurt initiation rate (in initiations per second).

Beta (β) is the talkspurt termination rate (in terminations per second).

α^{-1} is the mean length of a silent period (in seconds).

β^{-1} is the mean time (in seconds) of talkspurt duration.

$\alpha/(\alpha + \beta)$ is the proportion of time that a call in progress of infinite duration is active.

$\beta/(\alpha + \beta)$ is the proportion of time that a call in progress of infinite duration is silent.

2. Model Assumptions

It is assumed that calls are initiated in accordance with a Poisson process with mean rate λ. The length of a call in progress is exponential with mean μ^{-1}. Blocked calls (customers) are cleared; that is, new calls are prevented from initiation if all available channels are in use. Let $\{C(t);\ t \geq 0\}$ be the number of calls in progress at time t.

Calls in progress alternate between active and inactive states as talkspurts are initiated and terminated. We model this process as an alternating renewal process where the length of the talkspurt is exponential with mean β^{-1} and the length of a silent period is exponential with mean α^{-1}. Let $\{A(t);\ t \geq 0\}$ be the number of calls in progress that are active at time t. Note that $A(t) \leq C(t)$.

It is also assumed that when a new call is initiated, it is immediately active; that is, a talkspurt is simultaneously initiated. When a call terminates, it may do so from either an active or inactive state.

3. Description

The model is a two-dimensional birth-and-death queueing model. It maintains the Markov property inherent in one-dimensional birth-and-death queueing systems, i.e the system occupies "states," and the rates at which changes of state occur depend only on the instantaneous state of the system and not on the past history of the process. However, two variables are required to define the state space. The bivariate process $\{(C(t), A(t));\ t \geq 0\}$ is a continuous time Markov chain with the following:

$$
\begin{aligned}
&P\{C(t+h)=c,\ A(t+h)=a \mid C(t)=k,\ A(t)=j\} \\
&\qquad = [\lambda h + o(h)]\ I(j \le k)\ I(k < K) && \text{if} \quad c=k+1,\ a=j+1, \\
&\qquad = [(k-j)\alpha h + o(h)]\ I(j < k) && \text{if} \quad c=k,\ a=j+1, \\
&\qquad = [j\beta h + o(h)]\ I(j \le k)\ I(j > 0) && \text{if} \quad c=k,\ a=j-1, \\
&\qquad = [\mu j h + o(h)]\ I(k > 0)\ I(j > 0)\ I(j \le k) && \text{if} \quad c=k-1,\ a=j-1, \\
&\qquad = [\mu(k-j)h + o(h)]\ I(j > 0)\ I(j \le k) && \text{if} \quad c=k,\ a=j-1, \\
&\qquad = 0 && \text{otherwise,}
\end{aligned}
$$

$$
where\ I(x<y)=\begin{cases} 1 \text{ if } x<y \\ 0 \text{ if } x \ge y \end{cases}.
$$

A rate diagram for this model, where the maximum number of available channels is three, is shown below in Figure 1; see [Ref. 6:p. 360] for discussion of transition rate diagrams.

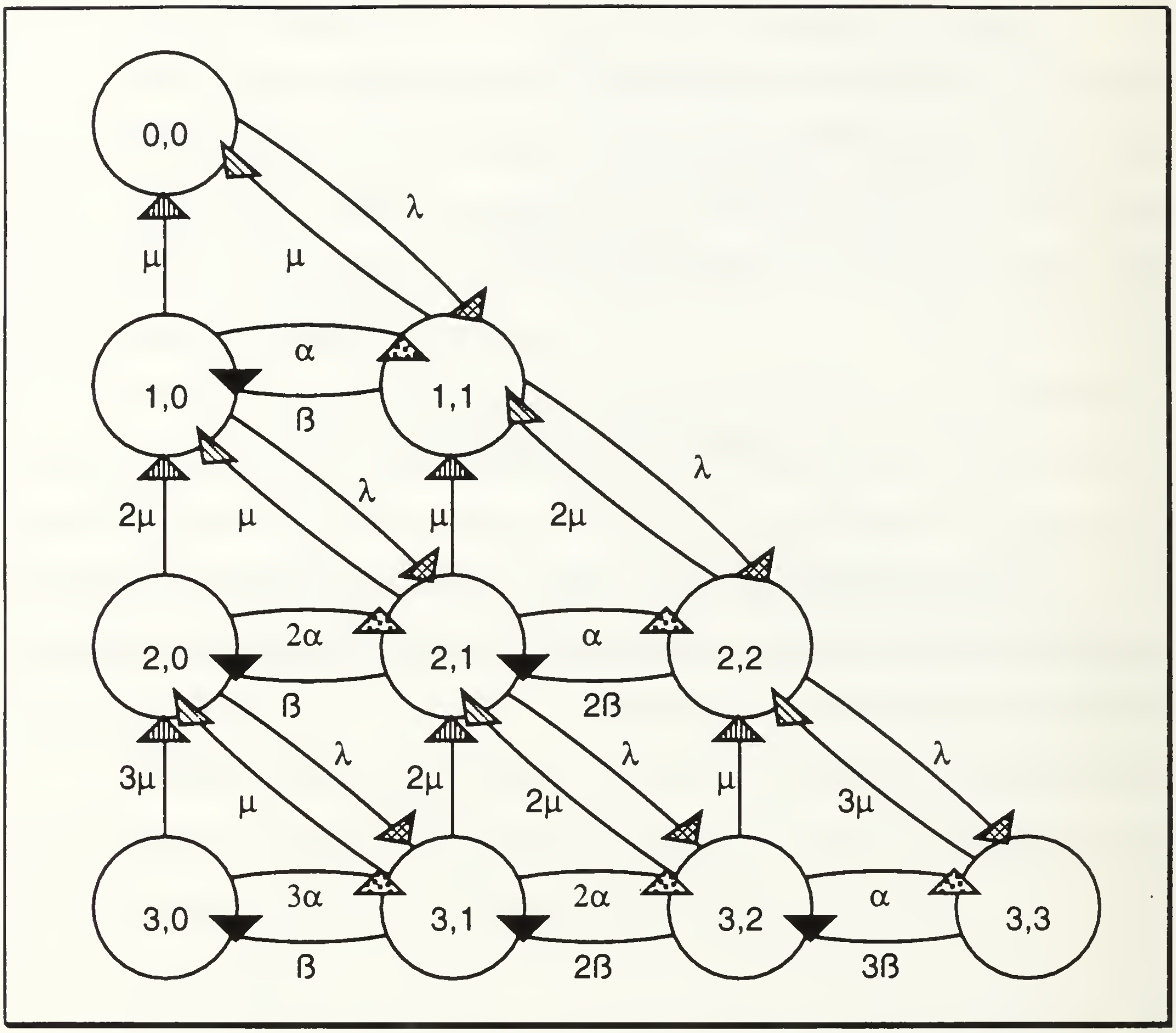

Figure 1. Graphical Representation of Three-Channel Bivariate Erlang System

Conservation-of-flow ("rate out = rate in") equations may be used to describe the system in equilibrium [Ref. 5:pp. 3-4]. We let the $\lim_{t\to\infty}P\{C(t) = k, A(t) = j\} = \Pi\{k,j\}$, where (k=0,1,..., K) and (j=0,1,...,k) represent the limiting distribution. The conservation-of-flow equations, which equate the rate the system leaves each state to the rate at which it enters that state, are shown below for a system with three available channels:

$$\begin{aligned}
\lambda\Pi\{0,0\} &= \mu\Pi\{1,0\} + \mu\Pi\{1,1\}, \\
(\alpha + \lambda + \mu)\Pi\{1,0\} &= \beta\Pi\{1,1\} + 2\mu\Pi\{2,0\} + \mu\Pi\{2,1\}, \\
(\beta + \lambda + \mu)\Pi\{1,1\} &= \lambda\Pi\{0,0\} + \alpha\Pi\{1,0\} + \mu\Pi\{2,1\} + 2\mu\Pi\{2,2\}, \\
(2\alpha + \lambda + 2\mu)\Pi\{2,0\} &= \beta\Pi\{2,1\} + 3\mu\Pi\{3,0\} + \mu\Pi\{3,1\}, \\
(\alpha + \beta + \lambda + 2\mu)\Pi\{2,1\} &= \lambda\Pi\{1,0\} + 2\alpha\Pi\{2,0\} + 2\beta\Pi\{2,2\} + 2\mu\Pi\{3,1\} + 2\mu\Pi\{3,2\}, \\
(2\beta + \lambda + 2\mu)\Pi\{2,2\} &= \lambda\Pi\{1,1\} + \alpha\Pi\{2,1\} + \mu\Pi\{3,2\} + 3\mu\Pi\{3,3\}, \\
(3\alpha + 3\mu)\Pi\{3,0\} &= \beta\Pi\{3,1\}, \\
(2\alpha + \beta + 3\mu)\Pi\{3,1\} &= \lambda\Pi\{2,0\} + 3\alpha\Pi\{3,0\} + 2\beta\Pi\{3,2\}, \\
(\alpha + 2\beta + 3\mu)\Pi\{3,2\} &= \lambda\Pi\{2,1\} + 2\alpha\Pi\{3,1\} + 3\beta\Pi\{3,3\}, \\
(3\beta + 3\mu)\Pi\{3,3\} &= \lambda\Pi\{2,2\} + \alpha\Pi\{3,2\}.
\end{aligned}$$

The sum of the terms on the left-hand side (rates out) is equal to the sum of the terms on the right-hand side (rates in). Any one of these equations is, thus, redundant and may be ignored. The remaining equations, along with the normalization equation

$$\sum_{k=0}^{3}\sum_{j=0}^{k}\Pi\{k,j\} = 1,$$

uniquely determine the limiting probabilities.

4. Parameter Values

If the average length of a phone call (μ^{-1}) is taken to be 180 seconds (three minutes), then $\mu = 1 \div 180$. The length of a talkspurt (β^{-1}) must be shorter than the length of a phone call for the model to be reasonable. We also want to maintain the proper proportion between the length of talkspurts and silent periods. Speech activity ranges from 28% to 42% depending on cultural and language characteristics of the user population [Ref. 16:p. 1]. If voice conversations are assumed silent 60% of the time, then we need to have $\beta \div (\alpha + \beta) = 0.60$. The input value for λ is treated as variable; increasing the value of λ corresponds to an increasing load on the system, where load is defined to be $\lambda \div \mu$. Increasing the load increases the blocking probability. The maximum number of channels is also treated as variable. Increasing the number of channels decreases the blocking probability.

B. THE MULTIPLEXER MODEL

The multiplexer model begins with the Erlang model as described above, then adds the three main features which are characteristic of how a multiplexer functions. The first and most important distinguishing characteristic of the multiplexer is that it allows more calls in progress than the actual physical number of channels. This is accomplished by taking advantage of the silent periods in each conversation to merge together packetized talkspurts from multiple conversations. Secondly, it compresses the packetized talkspurt to a fraction of its original length. Third, and lastly, it appends header information to each packet, to allow the talkspurt to be recreated at the destination node. See [Ref. 17:p. 430] for additional discussion of the information contained in the packet header.

1. Variables

The following are additional variables that appear in the multiplexer model. A new variable (J) is added, and the value of K is redefined. Also, β^{-1} is replaced by $(\beta^{-1})^*$, and service rate (s) is added.

J is the maximum number of talkspurts allowed (equal to the number of physical channels).

K is the maximum number of calls allowed to be in progress (may be several times greater than J).

$(\beta^{-1})^*$ is the new mean talkspurt length in units of bits per talkspurt after compression and addition of packet headers.

b is the number of bits per second produced by the coding scheme.

s, the service rate in bits per second, is simply the outgoing channel rate (of the wideband channel).

$\beta^* s$ is the new departure or service rate of talkspurts (in talkspurts per second), where β^* is the inverse of $(\beta^{-1})^*$.

2. Additional Model Assumptions for the Multiplexer Model

Although more calls than channels are allowed, new calls are blocked when the number of active calls in progress (talkspurts) equals the number of available channels. Voice packets belonging to a call in progress are also blocked (lost or "clipped") when the number of active calls in progress equals the number of available channels.

3. Description

In the multiplexer, all talkspurts from all incoming channels flow through a buffer, where they are "packetized" and sent forward along a single wideband channel. The multiplexer divides talkspurts into fixed size packets and attaches certain header information that allows the talkspurt to be reconstructed at the destination node by a demultiplexer. The multiplexer can also compress the packetized information so that it uses fewer bits, thus occupying less space as it moves through the channel. Typical compression schemes use either a 2-to-1 or 4-to-1 rate of compression.

The intelligent multiplexer model is also a bivariate process $\{(C(t), A(t)); t \geq 0\}$ and a continuous-time Markov chain with the following:

$$
\begin{array}{lll}
P\{C(t+h)=c,\ A(t+h)=a \mid C(t)=k,\ A(t)=j\} & & \\
\quad = [\lambda h + o(h)]\ I(j < J)\ I(k < K) & \text{if} & c=k+1,\ a=j+1, \\
\quad = [(k-j)\alpha h + o(h)]\ I(j < J)\ I(j < k \le K) & \text{if} & c=k,\ a=j+1, \\
\quad = [j(\beta^{*}s)h + o(h)]\ I(0 < j < J)\ I(k \le K)\ I(j \le k) & \text{if} & c=k,\ a=j-1, \\
\quad = [\mu jh + o(h)]\ I(k > 0)\ I(0 < j \le k \le K)\ I(j < J) & \text{if} & c=k-1,\ a=j-1, \\
\quad = [\mu(k-j)h + o(h)]\ I(0 < j < J)\ I(j < k < K) & \text{if} & c=k,\ a=j-1, \\
\quad = 0 & \text{otherwise,} &
\end{array}
$$

$$
\textit{where } I(x<y)=\begin{cases} 1 \text{ if } x < y \\ 0 \text{ if } x \ge y \end{cases}.
$$

A rate diagram for the multiplexer model, where the maximum number of available channels is three, is shown below in Figure 2; see [Ref. 6:p. 360] for discussion of transition rate diagrams.

In the multiplexer model, there can be two types of blocking. Outside calls can be blocked from initiation (external blocking) and calls in progress can be blocked from transmitting a talkspurt (internal blocking). Both kinds of blocking occur when the number of talkspurts (active calls) is at the line capacity

$$
\lim_{t\to\infty} P\{A(t)=J\} = \sum_{k=J}^{K} \lim_{t\to\infty} P\{C(t)=k, A(t)=J\}.
$$

The blocking of calls from initiation also occurs when the number of calls in progress is at the maximum allowed (C(t)=K). The proportion of time this occurs is given by

$$
\lim_{t\to\infty} P\{C(t)=K\} = \sum_{j=0}^{J} \lim_{t\to\infty} P\{C(t)=K, A(t)=j\}.
$$

In comparison, blocking in the Erlang model occurs only when the number of calls in progress equals the number of physical channels. There is

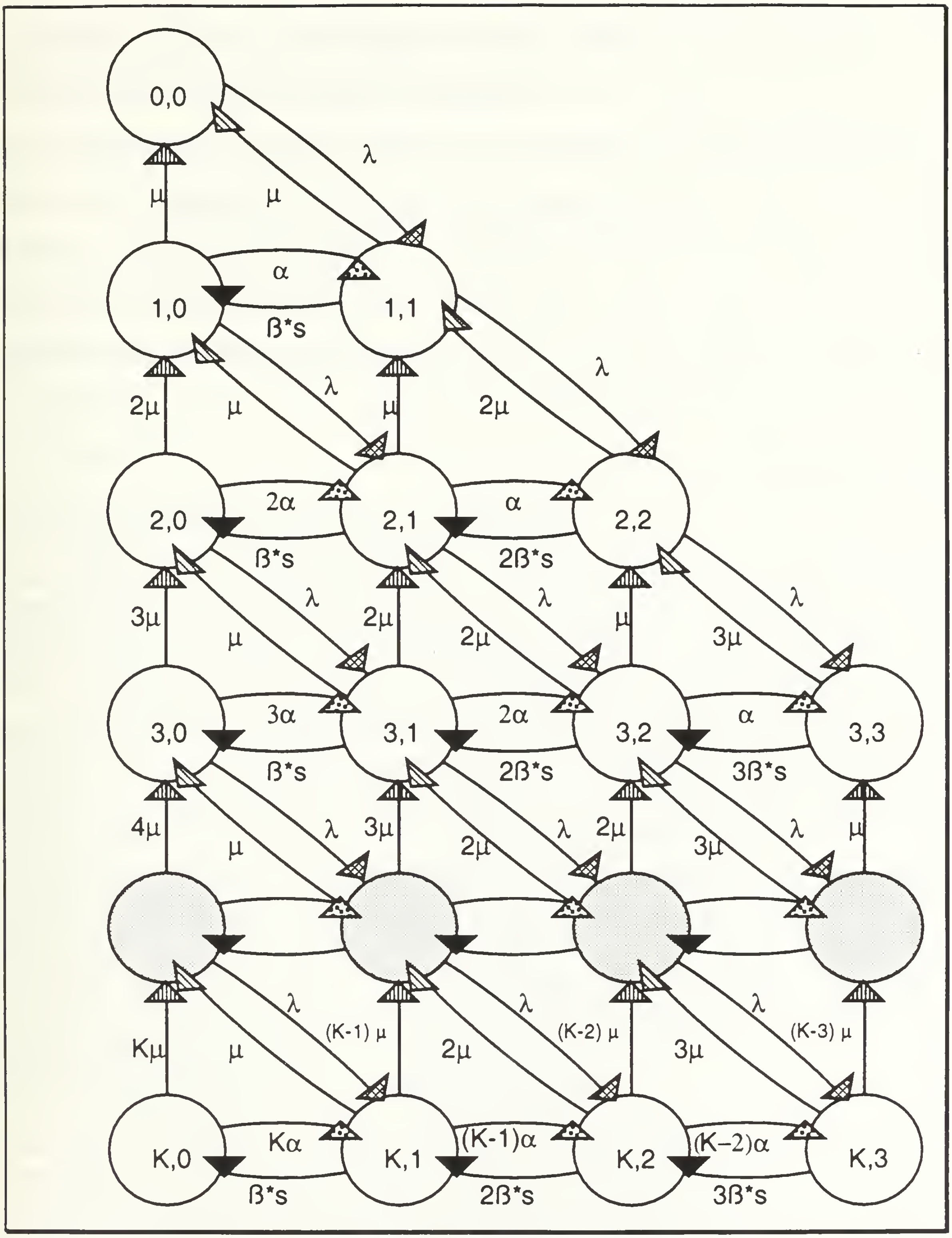

Figure 2. Graphical Representation of Three-Channel Smart Mux Model

no internal blocking. Note that under reasonable loading it is possible for $A(t) \leq J \leq C(t) \leq K$, where J is the maximum number of active calls that the transmission line can support and K is the maximum number of calls allowed in the system. For purposes of this paper, we will refer to the external blocking that occurs when $C(t)=K$ as outer blocking. The internal and external blocking that occurs when $A(t)=J$ will be called inner blocking. By adding the two together and subtracting out the joint limiting probability that $\{C(t)=K, A(t)=J\}$, we get the total probability of blocking.

D. PARAMETER VALUES

The value for length of talkspurts (β^{-1}) in the Erlang model changes in the multiplexer case to account for both compression of the packetized talkspurt and for header information appended to each packet. Packet lengths are expressed in terms of bits rather than time, but can be converted to units of time if given the line rate of the transmission medium in terms of bits per second (bps). The voice packet size depends on the coding scheme used. For 32 Kbps, ADPCM coding, and a packetization period of T=16 milliseconds (ms), the packet size is 512 bits or 64 bytes (there are 8 bits per byte), plus a header [Ref. 16:p. 1]. A talkspurt of 352 ms is divided into $352 \div 16 = 22$ packets and contains a total of 11264 bits (1408 bytes). Each packet is then compressed. A compression factor of four reduces each packet to 128 bits. Appending a packet header of 10 bytes to each compressed packet increases the length to 208 bits (26 bytes). Thus the number of bits in a talkspurt of 352 ms is 4576 after compression and addition of headers. This compression and addition of packet headers to alter the original mean talkspurt length, β^{-1} (in units of seconds), results in the new mean talkspurt length in units of bits, $(\beta^{-1})^*$, defined in the multiplexer model as follows:

$$(\beta^{-1})^* = \beta^{-1} \times T^{-1} \times (\text{\# bits/packet}) \times ((1 + \text{compress}) + \text{header proportion})$$

$$= \beta^{-1} \times b \times ((1 + \text{compress}) + \text{header proportion})$$

$$= \text{number of bits per average talkspurt},$$

where b, the number of bits per second produced by the coding scheme, is equivalent to the number of bits per packet (e.g. 512) divided by the packetization period T (e.g. 16 ms per packet). Also note that $\beta^{-1} \times T^{-1}$ is equal to the mean number of packets in a talkspurt.

Compress is set equal to four (4) to indicate a 4-to-1 compression of data by the multiplexer. Packet header information is assumed to be 10 bytes (attached to a 64 byte packet), [Ref. 16], for a header proportion of 10÷64 = .15625.

In addition, the service rate of the outgoing channel is now many times larger than any of the incoming channels. The Defense Communications Agency commonly uses T1 lines, which carry 1.544 Mbps (1.536 Mbps after accounting for the signalling channel). The T1 lines may be divided into 1.536 Mbps÷32 Kbps = 48 separate channels. Therefore the outgoing T1 rate is 48 times larger than the rate of the encoding scheme. A talkspurt of 352 ms (without compression and addition of packet header) will take 11264 bits÷1.536 Mbps = 0.073 ms to transmit on a T1 line.

In this multiplexer model, however, we do not necessarily want to assume full T1 rates for the outgoing channel. Rather, we need to be able to look at fractional T1 rates for lighter traffic loads, so we assume that the outgoing rate is equal to 32 Kbps multiplied by the maximum number of active calls allowed (labeled J in the multiplexer model described above; labeled A in the computer code). The incoming channel rate is set equal to b = 32 Kbps. The ratio of the outgoing channel rate to the rate of an active incoming channel is set equal to J. In the multiplexer model, J × b is defined as the service rate, s. The termination rate for talkspurts in the multiplexer model is given by $\beta^* s$.

V. APPROXIMATIONS

A. THE ERLANG MODEL APPROXIMATION

As noted earlier, the truncated Poisson formula is used to calculate the limiting probabilities for an Erlang loss system with maximum K channels and input parameters λ and μ; that is, a model for the calls in progress $\{C(t); t \geq 0\}$ is a continuous time Markov chain with transition rate diagram shown in Figure 3 [Ref. 6:p. 360].

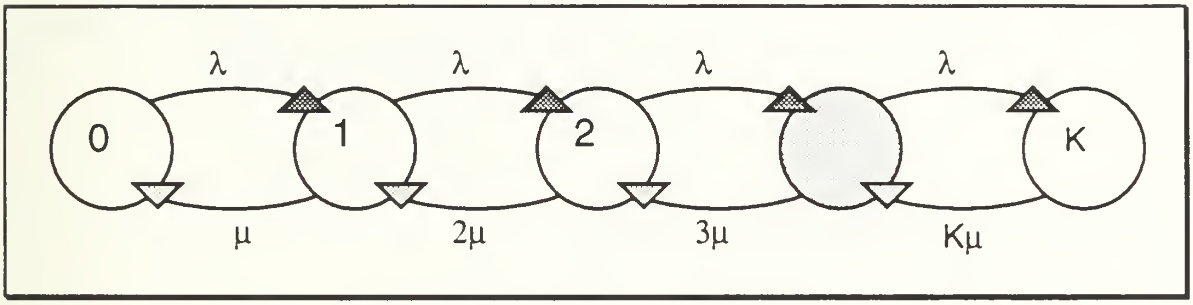

Figure 3. Transition Rate Diagram for Calls in Progress

To deal with the bivariate Erlang system, we need to consider the two additional parameters (α and β) which describe talkspurt initiation and termination. Fix the number of calls in progress equal to $k \leq K$. A model for the number of active calls in progress is a continuous time Markov chain with the rate diagram shown in Figure 4. Since the calls in progress are independent of each other, the limiting distribution of having j active calls is described by the binomial distribution;

$$\lim_{t \to \infty} P\{A(t) = j | k \text{ calls in progress}\} = \binom{k}{j}\left(\frac{\alpha}{\alpha+\beta}\right)^{j}\left(\frac{\beta}{\alpha+\beta}\right)^{k-j}. \qquad (4)$$

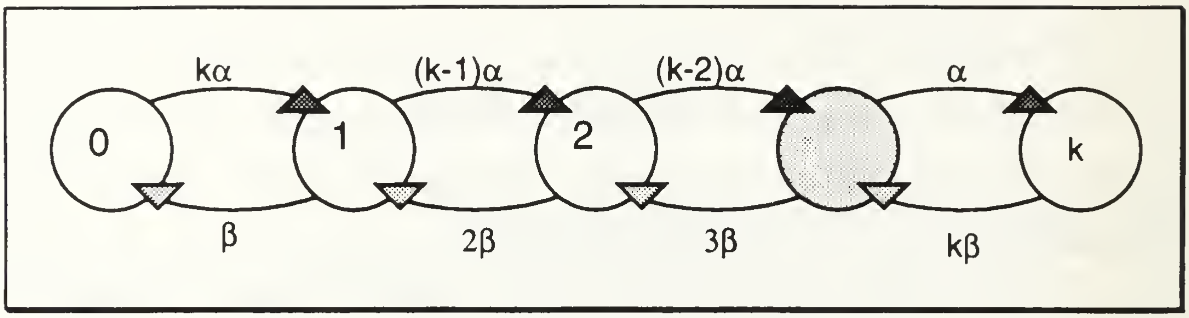

Figure 4. Transition Rate Diagram for Active Calls in Progress

Limiting probabilities for the bivariate Erlang system can be approximated by combining the truncated Poisson distribution (1) with the binomial;

$$\lim_{t\to\infty} P\{A(t)=j, C(t)=k\} = \frac{\dfrac{(\lambda/\mu)^k}{k!}}{\displaystyle\sum_{i=0}^{K}\frac{(\lambda/\mu)^i}{i!}} \binom{k}{j}\left(\frac{\alpha}{\alpha+\beta}\right)^j\left(\frac{\beta}{\alpha+\beta}\right)^{k-j}, \qquad (5)$$

where (k = 0,1,...., K) and (j= 0,1,...., k).

B. THE MULTIPLEXER MODEL APPROXIMATION

For the multiplexer model, the binomial probability of having j talkspurts, given k calls in progress, must be adjusted to reflect the new restriction that the number of talkspurts cannot exceed the number of physical channels J, and that J may be less than k. The following form of the truncated binomial [Ref. 5:p. 109] was used rather than the binomial distribution used in the Erlang model.

$$P_j(k) = \lim_{t\to\infty} P\{A(t)=j \mid k \text{ calls always in progress}\} = \binom{k}{j}\left(\frac{\alpha}{\beta * s}\right)^j P_0(k), \qquad (6)$$

$$\text{where } P_0(k) = \left[\sum_{j=0}^{J}\binom{k}{j}\left(\frac{\alpha}{\beta * s}\right)^j\right]^{-1}$$

for j ≤ k, where (k = 0, 1,...., K) and (j= 0, 1,...., J).

The truncated Poisson distribution (1) is still used to find the probability of k calls in progress (k=0,1...,K), but now it yields an approximate rather than an actual limiting probability, since it fails to account for the additional internal blocking in the multiplexer model. Thus, the truncated Poisson yields a conservative estimate of the external blocking that occurs when the maximum allowed number of calls are in progress (outer blocking).

The joint approximate limiting probabilities for the multiplexer model are similarly found by multiplying the truncated Poisson by the truncated binomial; that is,

$$\lim_{t\to\infty} P\{C(t)=k, A(t)=j\} = \left[\frac{\frac{(\lambda/\mu)^k}{k!}}{\sum_{k=0}^{K}\frac{(\lambda/\mu)^k}{k!}}\right]\left[\frac{\binom{k}{j}\left(\frac{\alpha}{\beta * s}\right)^j}{\sum_{j=0}^{J}\binom{k}{j}\left(\frac{\alpha}{\beta * s}\right)^j}\right], \tag{7}$$

for j ≤ k, where (k = 0,1,...,K) and (j = 0,1,...,J).

VI. SOLUTION TECHNIQUES

A. SOLVING SETS OF LINEAR EQUATIONS

Two programs were written to solve the system of linear equations determining the limiting distribution (for both the Erlang and the multiplexer models). One uses GAMS [Ref. 18], which is a software package developed to solve large mathematical (linear and non-linear) programming models. The other uses APL to solve the system of equations through matrix inversion and was developed by Professor Patricia Jacobs of the Naval Postgraduate School. The GAMS programs may be found in Appendix A (Erlang model) and Appendix B (multiplexer model). The APL program for the multiplexer model, in Appendix C, may also be used to solve the Erlangian system with some adjustments to the input variables.

This solution technique, though accurate, was found to be useful only for small problems. Using an IBM mainframe computer, the GAMS programs were solvable for systems of about 15 channels in the Erlang model (with a load of 15 Erlangs). Beyond that, the solver encounters overflow problems. For discussion of the computational instability of solving the matrix equations and alternative solution techniques, see Anick, Mitra, and Sondhi [Ref. 7:pp. 1873-1874]. The APL programs MATRIXE and MATRIXM were solved using APL2 on an IBM mainframe. Without increasing the workspace size beyond the default, it is possible to solve for systems of up to size 21 × 21; that is, where 21 is the number of both the maximum number of calls in progress and the maximum number of active calls in progress allowed (253 states). It is possible to increase the size of the workspace from the default of 65% to a maximum of 85%, and thereby increase the size of the matrix which can be solved. However, it takes a long time to solve the larger systems, especially when creating tables of multiple runs.

B. APPROXIMATION

The approximation routine APPROX, written in APL, calculates the limiting probabilities for both the Erlang and the multiplexer models. It may be found in Appendix D. The approximation routine is much faster than solving the sets of linear equations required to find the limiting distribution of the bivariate models. It is also able to solve larger problems, given the same APL workspace size. On the IBM mainframe APPROX can solve problems up to size 32 × 32 (561 states) before encountering underflow errors in the results (due to extremely small limiting probabilities, on the order of $1E^{-}75$ or smaller). The approximation will solve for systems of up to $C = 175$ (maximum calls in progress allowed) without halting due to domain errors (numbers larger than 1E75 in the intermediate calculations). Results from these larger systems may, however, be inaccurate due to the underflow errors mentioned above, depending on the value of A (number of physical channels). For instance, when solving for a system with C equal to 40, the approximation was able to calculate the results for as many as $A = 33$ channels before encountering underflow errors.

C. VALIDATION OF THE COMPUTER CODE

1. Validating Code for the Erlang Model

The computer code was validated in two ways. First the results for one, two and three-channel systems were calculated by hand for a particular set of values for λ, μ, α and β to ensure that results matched those of the computer programs. Second, numerous cases were calculated using both the APL (MATRIXE) and the GAMS (ERLANG) programs to ensure that the two different programs yield the same results. The APL (APPROX) program for the Erlang model was then compared with results from APL (MATRIXE) to ensure that the approximation routine yields results which are close to the actual limiting probabilities.

2. Validating Code for the Multiplexer Model

The multiplexer codes (MUX in GAMS and MATRIXM in APL) were first validated by ensuring they yield the same results as the Erlang codes (ERLANG in GAMS and MATRIXE in APL) when all the same parameter values are used as inputs (i.e. no change in the service rate, no compression or packet header, and the number of channels J equals the maximum number of calls allowed K). The APL (MATRIXM) and GAMS (MUX) programs were also compared to each other to ensure the same results for various sets of input parameters. Results were also checked for internal consistency; that is, individual input parameter values were changed separately to check that the output values change as expected. Finally, the results of the APL (APPROX) program for the multiplexer model were compared with those of the APL (MATRIXM) program to check the validity of the approximation routine and determine the range of values over which the approximation yields results close enough to be used as a tool in determining the reduction in bandwidth requirements.

VII. NUMERICAL RESULTS

A. ACTUAL VS. APPROXIMATED BLOCKING PROBABILITIES

Results of several comparisons between the actual (MATRIXM) and approximated (APPROX) multiplexer model are shown in Appendix E. Comparisons were made for systems allowing a maximum of C = 5, 10, 15, 20, and 30 callers, assuming speech activity (average proportion of time a call in progress is active) of 35%. Traffic loads displayed depend on the value for C; the larger the value for C, the heavier the loads, though not larger than the value for C itself. This restricts the results, and analysis of those results, to the range of values for blocking probabilities which might be considered reasonable to plan for when designing a telecommunications system.

The results shown in Appendix E indicate that the approximated outer blocking (OUTBLA) becomes very close to the actual value (OUTBL) as the gap between A (number of channels) and C (maximum number of calls allowed) decreases. In fact, when A equals C, OUTBL and OUTBLA are also equal. The approximated inner blocking (INBLA) also becomes closer in value to actual inner blocking (INBL) as A and C become closer. The probability of inner blocking decreases, becoming zero when A equals C. The size of the limiting probability of inner blocking is, therefore, also closely linked to the difference between the actual and approximated outer blocking probabilities. As inner blocking decreases, OUTBLA becomes closer to the actual values. Note that there is a trade-off between outer and inner blocking.

Inner blocking increases as the gap between A and C increases, while outer blocking decreases.

The question is, at what point are the approximations close enough to the actual values to be used to determine limiting probabilities; that is, how close does A need to be to C ? For inner blocking probabilities, the approximation results are extremely close to the actual values for even large relative gaps between A and C. For instance, when C = 5, 10, 15, 20, and 30, INBLA is accurate to 3 decimal places when A = 2, 3, 4, 4, and 5, respectively (for all traffic loads displayed). Also, when INBLA is accurate to 3 decimal places, the first 2 decimal places hold zeros. For the same values of C and the same traffic loads, OUTBLA is accurate to approximately 2 decimal places for A = 3, 3, 4, 4, and 5, respectively. Thus, INBLA is somewhat more accurate than OUTBLA and the size of the values for INBLA may be a good predictor of the accuracy of both INBLA and OUTBLA. Suppose we develop a 'thumb rule' that states: when INBLA is equal to zero in the first 'x' decimal places, (a) INBLA is accurate to within 'x+1' decimal places, and (b) OUTBLA is accurate to within 'x' decimal places. Close examination of the results in Appendix E indicate that our thumb rule is accurate for all values of C, A, and load shown, if the values for OUTBL are rounded to 'x' decimal places for comparison with OUTBLA. Thus, by using the approximated inner and outer blocking together, we can tell fairly accurately how close (within number of decimal places) OUTBLA is to the actual outer blocking probability by looking at the proportion of inner blocking.

As to answering the question posed, i.e. how close must A be to C for accurate results, the response depends on two things; (1) the level of accuracy desired, and (2) the value of C. For telephone traffic engineering purposes, the level of accuracy necessary is generally 2 or 3 decimal places, so we want the values for INBLA to have zeros in at least the first 2 decimal places. Clearly, the ratio of A to C necessary for accurate results decreases as C gets larger.

Having developed a thumb rule methodology for determining the accuracy of the multiplexer approximation results without direct comparison with actual values, we may now look at the results of the approximation independently, allowing analysis of larger systems. The approximate results are much more quickly obtained, making it feasible to conduct multiple runs for different levels of speech activity. Analysis of these results, displayed in Appendix F, is the subject of the next section.

B. SENSITIVITY ANALYSIS OF THE APPROXIMATED INNER BLOCKING PROBABILITIES

The approximation routine for the multiplexer model was run for different values of the initial mean length of a talkspurt, β^{-1}, and mean length of a silence, α^{-1}, such that speech activity occupies 28 percent, 35 percent, and 42 percent of a call in progress. This was to determine sensitivity of the inner blocking probabilities (4) to changes in speech characteristics. Since the approximated outer blocking probability is calculated from the Erlang loss formula (2), it is not affected by any parameters other than λ, μ, and K.

The average length of a phone call, μ^{-1}, was taken to be 180 seconds (3 minutes) for all runs. Speech activity rates considered were 28, 35 and 42 percent. The mean talkspurt and silence lengths are assumed to be 288 ms and 740 ms for the first case, 352 ms and 650 ms for the second case, and 420 ms and 580 ms for the third case, respectively. Values for the last two cases are the same as those used by Sriram and Lucantoni [Ref. 14:pp. 703-712].

Results of runs for C = 5, 10, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 125, 150, and 160 are given in Appendix F. To use the table in Appendix F, you first find the load (column 1) for which outer blocking probability (column 2) is less than or equal to a specified value, say 0.01. In the case of C = 20, the corresponding load is 12. The next three columns give the approximate inner blocking probability for speech activity rates of 28, 35, and 42 percent, respectively. The most conservative (highest) estimate of inner blocking would, of course, be found in the last column, representing the 42% activity level. If you wish a total blocking probability of no more than 0.01, accurate to within 2 decimal places, then you find the value of A for which, given a load of 12, the value for inner(42) is zero in at least the first 2 decimal places, and the addition of the outer and inner(42) blocking probabilities is closest to, but still no greater than, 0.01. Notice that we are not subtracting out the joint blocking probability (as on page 21) after adding together the inner and outer blocking probabilities. This is primarily because the joint blocking probabilities are so small as to be insignificant to the results of the calculations. Also, any error thus induced would be on the side of conservatism, and therefore tolerable. For this example, the value for A (number of channels) which meets the requirement is 5, which is one-fourth of the value for C (maximum number of callers).

Figure 5 shows a graphical representation of the data from Appendix F, for C = 20 callers and speech activity of 35%. It actually represents two graphs superimposed on each other. The one graph shows outer blocking probability versus load when C (maximum number of callers allowed) is equal to 20. This is calculated using the Erlang loss formula (2). Curves for C<20 would be higher and to the left of the curve for C=20 (+ symbol); curves for C>20 would be lower and to the right. Graphs showing the curves for selected values of C ranging from 1 to 80 may be found in Cooper [Ref. 5, pp. 316-319]. Cooper uses different symbols and also uses a logarithmic scale for the blocking probabilities, which gives a different shape to the curves. The calculations and results, however, are the same. The other graph displayed in Figure 5 is inner blocking probability versus load for various values of A (A = 3, 4, 5, 6) when C=20 and speech activity is 35%. Remember that the value for A represents the number of channels (or equivalent bandwidth) available. The goal is to minimize the value of A while maintaining a specified standard of service; in this case, total probability of blocking no greater than .01.

From Appendix F we see that when C=20 and the load is 12 erlangs, the outer blocking probability equals .009796, and 12 is the highest load the system can take without exceeding the .01 limit on total blocking. Inner blocking can be no greater than .000204. We must find the value for A which satisfies this requirement. For speech activity of 35%, A=5 channels is sufficient, with inner blocking of .000148. Three channels is clearly too few, four channels will only work at the 28% level of speech activity, and six channels exceeds the standard of service required.

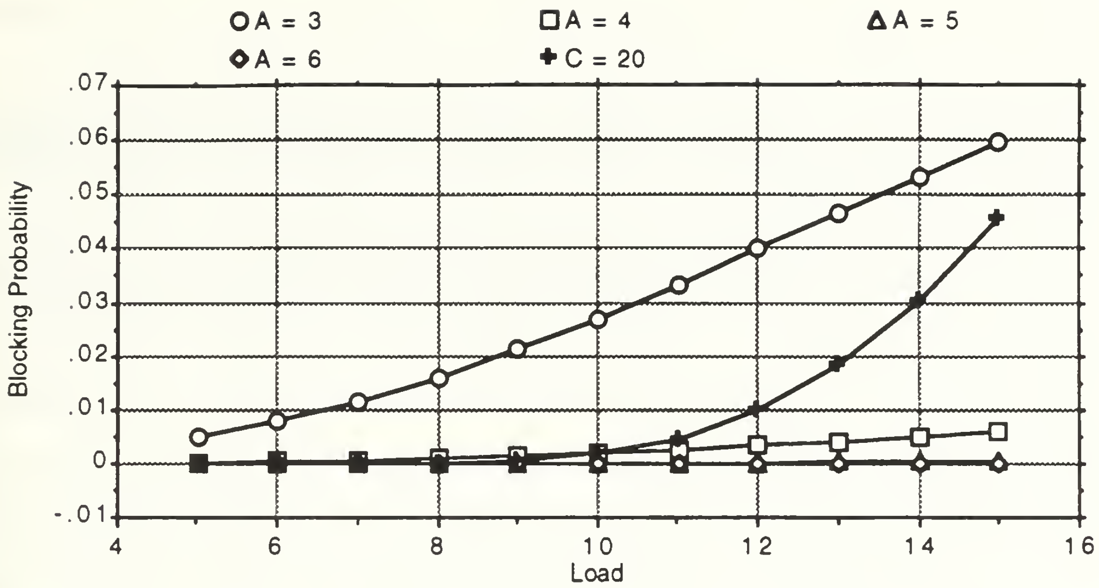

Figure 5. Outer Blocking Probability vs. Load for C = 20, shown with Inner Blocking Probability vs. Load for various values of A (given C = 20), assuming 35% speech activity. From Appendix F.

Given specific criteria for desired blocking probabilities and accuracy levels, we can make tables of the values for the load and for A necessary to meet those criteria for each value of C. Conversely, if the load is fixed, there is a specific value for C which will meet the desired blocking probability. We can also determine the magnitude of the effect that the proportion of speech activity has on the value of A chosen. Table 1 below is an example, where the desired total blocking probability (again ignoring joint blocking) is no greater than 0.01 and is accurate to within three decimal places. The data from Table 1 are graphically depicted in Figures 6 through 8.

TABLE 1. VALUES OF A FOR GIVEN LEVELS OF SPEECH ACTIVITY, WITH TOTAL BLOCKING NO GREATER THAN 0.010; ACCURATE TO 3 DECIMAL PLACES.

C	LOAD	A: INNER (28)	A. INNER (35)	A. INNER (42)
5	3	3	3	4
10	4	3	3	4
20	12	4	5	5
25	16	5	5	5
30	20	5	5	5
35	24	5	5	6
40	29	6	6	7
45	33	6	6	7
50	37	6	6	7
60	46	6	7	7
70	56	7	8	8
80	65	7	8	9
90	74	7	8	9
100	84	8	9	10
125	107	8	9	10
150	131	9	10	11
160	141	9	11	12

Results of this study indicate that for low loads, the addition of multiplexers provides very little, if any, advantage in terms of reducing the number of channels necessary to provide acceptable blocking probabilities. The advantage increases dramatically as load increases. This is shown in Figure 6, where C and A represent the number of channels needed without and with multiplexers, respectively. Also, the level of speech activity does have some impact on the number of channels required. However, the values of A for 35% speech activity are within ±1 channel of the values obtained for the lower (28%) and upper (42%) speech activity levels. This is shown in Figure 7, which gives a closer view of the bottom three lines from Figure 6. Figure 8 shows the use of regression analysis to interpolate the number of channels required for loads between those listed. The quadratic equation generated by the regression gives a model for predicting the value of A (on the Y axis) when the load (on the X axis) is known, given desired total blocking of no greater than 0.01 (accurate to within 3 decimal places) and speech activity of 35%. Note that since the information in Figures 6 through 8 is taken from Table 1, all three figures assume total desired blocking probabilties of .01. Once this is fixed, it fixes the value of C for every corresponding load, and vice versa. Therefore, the values given for A are dependent on the value of C as well as on the load, and C could be substituted for load on the X axis of the three graphs. The fact that load and C are dependent on each other allows us to use just the load to determine the value of A (number of channels needed for a multiplexed system) without doing the intermediate calculation to find the value of C (number of channels required for a non-multiplexed system), given, of course, that we know the desired total blocking probability and level of accuracy required.

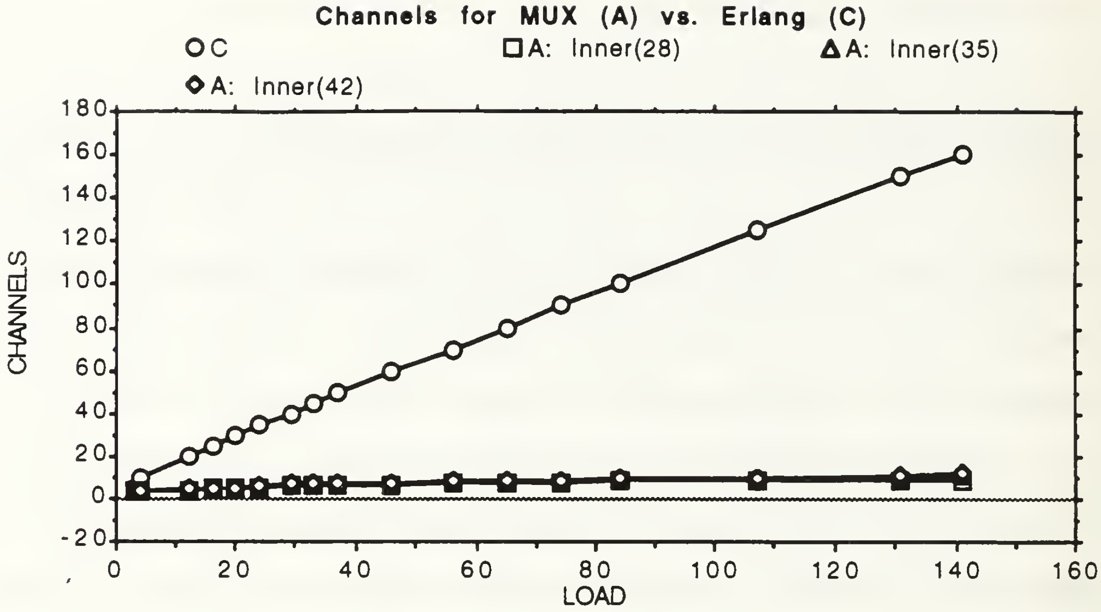

Figure 6. Channel Reduction in the Multiplexer Model

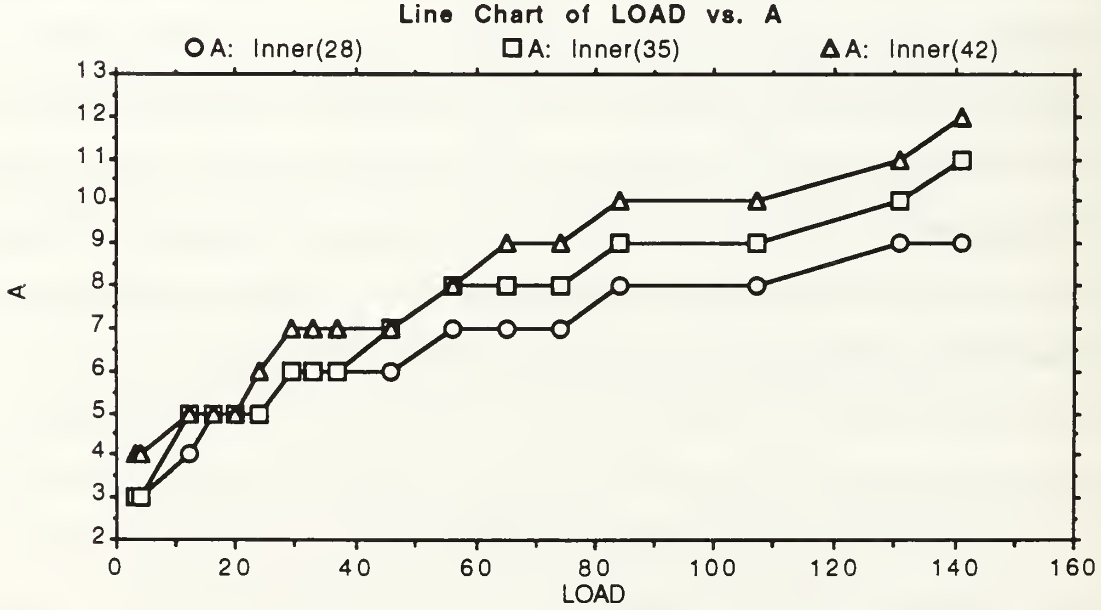

Figure 7. Channels Required for Various Speech Activity Levels; Mux Model

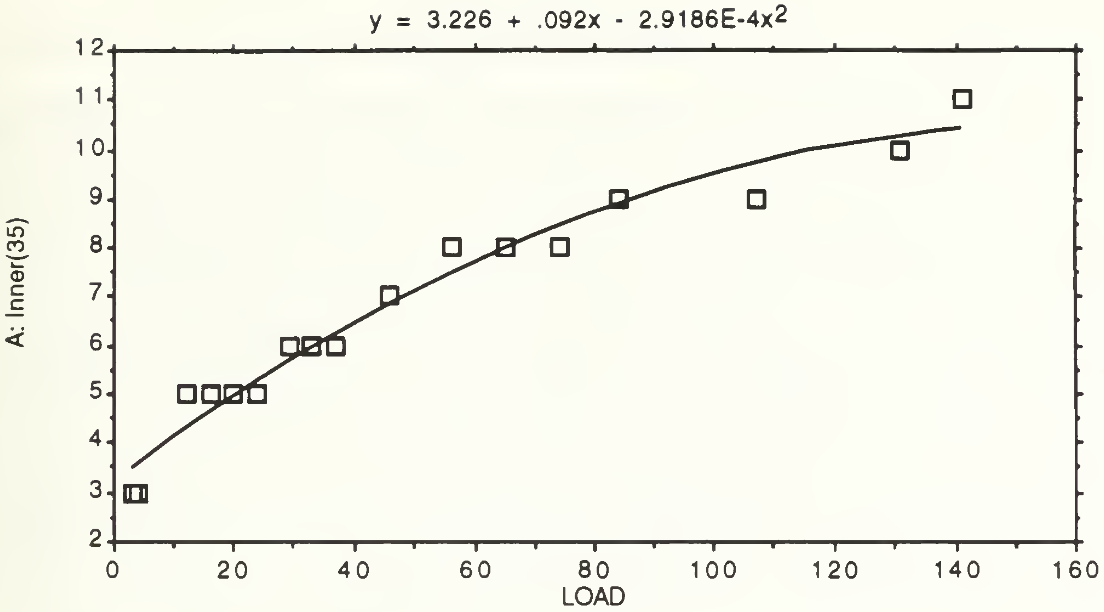

Figure 8. Regression on Channels (A) for Mux Model vs. Load (35% speech activity)

VIII. CONCLUSIONS

In this study, we first developed stochastic models of a single-node telecommunications system both without and with the addition of an intelligent multiplexer (the bivariate Erlang and the Multiplexer models, respectively). The models were solved using matrix equations to compute the joint limiting probabilities for k callers and j talkspurts, as well as outer blocking and inner blocking probabilities (respectively the proportion of time the maximum allowed numbers of callers and talkspurts are in the system). Both GAMS (Appendices A and B) and APL (Appendix C) were used to do the computations for the purpose of validating the computer code.

Approximation routines (Appendix D) were then developed that were capable of performing the calculations much faster and for larger systems. Results from the multiplexer approximation were compared with the actual blocking probabilities computed from the matrix equations (Appendix E). A rule of thumb based on the size of approximate inner blocking probabilities was devised to determine the accuracy of both the approximate inner and outer blocking probabilities. Sensitivity analysis was also done to determine the effect of different levels of speech activity on the inner blocking probabilities (Appendix F). Given desired outer blocking and total blocking probabilities, as well as desired level of accuracy, it is possible to determine the number of channels (A) required to handle a particular traffic load in the multiplexer model, and compare this with the number of channels (C) required in the Erlang model.

Analysis of the results from the tables in Appendix F indicate that addition of a multiplexer significantly reduces bandwidth requirements, particularly for heavy loading. The multiplexer advantage decreases to the point of insignificance as the load becomes very small (less than 3). The point at which the addition of multiplexers becomes advantageous depends on the cost of the adding multiplexers to a network vs. the cost of leasing the additional channels or bandwidth. These costs are affected by the number of nodes in a network, the geographical distances between nodes, and the loading along the links between nodes. A lightly loaded network with many nodes which are close together will benefit less than a heavily loaded system with long distances between relatively few nodes.

This study does not compare model results with data from actual systems. Nor was the multiplexer model developed to fit data from a real system. The Erlang loss formula has been found to have much practical use in designing voice telecommunications systems which do not utilize intelligent multiplexers. It is hoped that the methodology employed to adapt the bivariate Erlang model to reflect particular multiplexer characteristics will likewise prove useful in determining bandwidth requirements for systems which use intelligent multiplexers. Further study is recommended to validate the multiplexer model through comparison with data from a multiplexed voice system. Adjustments to the model may also be made to reflect different performance characteristics and input parameter values.

APPENDIX A

The following GAMS program computes limiting probabilities for the bivariate Erlang system. Results shown are for a three-channel system with the following characteristics:

Mean call length (μ^{-1}) of 3 minutes (180 seconds).

Load (λ/μ) equal to 1.

Mean talkspurt length (β^{-1}) of 352 ms (.352 seconds)

Mean length of silence (α^{-1}) of 650 ms.

Speech activity of 35% ($\alpha \div (\alpha+\beta) = 0.35$).

```
*------------GAMS AND DOLLAR CONTROL OPTIONS-----------------------
*                   (SEE APPENDICE B & C)



OPTIONS
    LIMCOL =  0 , LIMROW  = 0    , SOLPRINT = OFF , DECIMALS = 6
    RESLIM =  200, ITERLIM = 20000, OPTCR = 0.1 , SEED = 3141;

*-------------DEFINITIONS AND DATA--------------------------------
  SET
     C   channels /0*3/;

  ALIAS (C,A);

  SCALARS
     alpha  talkspurt arrival rate in SEC(inverse) /1.53846/
     beta   talkspurt departure rate in SEC(inverse)  /2.8409/
     lambda  customer arrival rate in SEC(inverse) /.005556/
     mu      customer departure rate in SEC(inverse) /0.005556/
     totprob  /1/
     B      maximum number of channels /3/;

*-------------------HODEL-----------------------------------------
  POSITIVE VARIABLES
      P(C,A)    limiting probability that system is in that state
      ZP(C,A)   z-plus dummy variable
      ZM(C,A)   z-minus dummy variable;

  VARIABLE
      Z          objective function to be minimized;

  EQUATIONS
      OBJ            dummy variables
      PROB(C,A)      calculate limiting probabilities
      NULL(C,A)      delete infeasible state spaces
      TOTL           observe upper bound on total probability;

* minimize
  OBJ..
      Z =E= SUM((C,A)$(ORD(C) GE ORD(A)), ZP(C,A)+ZM(C,A));

* subject to
  PROB(C,A)$(ORD(C) GE ORD(A) AND ORD(C) GT 1)..
      ZP(C,A) - ZM(C,A) =E=
*
      -P(C,A)*
      (LAMBDA$(ORD(C) LE B) +
      ALPHA*(ORD(C)-ORD(A)) +
      BETA*(ORD(A)-1) +
      MU*(ORD(C)-1))+
*
      P(C,A-1)*ALPHA*(ORD(C)-ORD(A)+1)+
*
      P(C,A+1)*(BETA*ORD(A))+
*
      P(C+1,A)*(MU*(ORD(C)-ORD(A)+1))+
*
      P(C+1,A+1)*((MU*ORD(A))$(ORD(C) LE B))+
*
      P(C-1,A-1)*LAMBDA ;

  NULL(C,A)$(ORD(C) LT ORD(A))..
      P(C,A) =E= 0;

  TOTL..
      SUM((C,A)$(ORD(C) GE ORD(A)),P(C,A)) =E= TOTPROB;


  MODEL P1 /ALL/;
  SOLVE P1 USING LP MINIMIZING Z;

*--------------------REPORTS---------------------------
*print the optimal objective value and solution.
 DISPLAY Z.L, P.L;
```

```
A         0          1          2          3
C
0   0.374992
1   0.242957   0.132046
2   0.078704   0.085551   0.023248
3   0.016997   0.027713   0.015062   0.002729
```

APPENDIX B

The following GAMS program computes limiting probabilities for the multiplexer model system. Results shown are for a three-channel system with the following characteristics:

Maximum number of calls allowed equals 5.

Mean call length (μ^{-1}) of 3 minutes (180 seconds).

Load (λ/μ) equal to 1.

Mean talkspurt length (β^{-1}) of 352 ms (.352 seconds)

Mean length of silence (α^{-1}) of 650 ms.

Speech activity of 35% ($\alpha \div (\alpha+\beta) = 0.35$).

```
*------------GAMS AND DOLLAR CONTROL OPTIONS-----------------------
*                  (SEE APPENDICE B & C)



OPTIONS
    LIMCOL =  0  , LIMROW  = 0    , SOLPRINT = OFF , DECIMALS = 6
    RESLIM =  100, ITERLIM = 10000, OPTCR    = 0.1 , SEED     = 3141;

*-------------DEFINITIONS AND DATA----------------------------------
  SET
     C   cellers /0*5/
     A   talkspurts /0*3/;

  SCALARS
     alpha  talkspurt arrival rate in SEC(inverse) /1.53846/
     betam  talkspurt departure rate in SEC(inverse)  /6.993/
     s      service rate /3/
     lambda  customer arrival rate in SEC(inverse) /.005556/
     mu      customer departure rate in SEC(inverse) /0.005556/
     totprob  /1/
     K      maximum number of cells /5/
     J      maximum number of active cells /3/;

*--------------------MODEL------------------------------------------
  POSITIVE VARIABLES
      P(C,A)    limiting probability that system is in that state
      ZP(C,A)   z-plus dummy variable
      ZM(C,A)   z-minus dummy variable;

  VARIABLE
      Z           objective function to be minimized;

  EQUATIONS
      OBJ            dummy variables
      PROB(C,A)      calculate limiting probabilities
      NULL(C,A)      delete infeasible state spaces
      TOTL           observe upper bound on total probability;

* minimize
  OBJ..
      Z =E= SUM((C,A)$(ORD(C) GE ORD(A)), ZP(C,A)+ZM(C,A));

* subject to
  PROB(C,A)$(ORD(C) GE ORD(A) AND ORD(C) GT 1)..
      ZP(C,A) - ZM(C,A) =E=
*
      -P(C,A)*
      (LAMBDA$(ORD(C) LE K) +
      ALPHA*(ORD(C)-ORD(A))$(ORD(A) LE J)+
      BETAM*(ORD(A)-1)*S +
      MU*(ORD(C)-1))+
*
      P(C,A-1)*ALPHA*(ORD(C)-ORD(A)+1)+
*
      P(C,A+1)*(BETAM*ORD(A)*S)+
*
      P(C+1,A)*(MU*(ORD(C)-ORD(A)+1))+
*
      P(C+1,A+1)*((MU*ORD(A))$(ORD(C) LE K))+
*
      P(C-1,A-1)*LAMBDA ;

  NULL(C,A)$(ORD(C) LT ORD(A))..
      P(C,A) =E= 0;

  TOTL..
      SUM((C,A)$(ORD(C) GE ORD(A)),P(C,A)) =E= TOTPROB;

  MODEL MUX /ALL/;
  SOLVE MUX USING LP MINIMIZING Z;

*----------------------REPORTS--------------------------
*print the optimal objective value and solution.
 DISPLAY Z.L, P.L ;
```

```
A          0            1            2            3
C
0     0.368137
1     0.342869     0.025235
2     0.159666     0.023502     0.000865
3     0.049569     0.010945     0.000806     0.000020
4     0.011536     0.003396     0.000375     0.000018
5     0.002147     0.000790     0.000116
```

APPENDIX C

The following program may be used to solve limiting probabilities for the bivariate Erlang model by setting "COMPRESS" equal to 1, "RO" equal to B and "HEADER" equal to 0.

```
      ∇MATRIXM CD
[1]  ⍝ MATRIX FOR ADAPTIVE MULTIPLEXER
[2]   ⎕IO←1
[3]  ⍝ THIS PROGRAM USES MATRIX INVERSION TO COMPUTE
LIMITING
[4]  ⍝ PROBABILITIES FOR THE MULTIPLEXER MODEL.
[5]  ⍝ IT REQUIRES A VECTOR INPUT OF 8 ELEMENTS.
[6]  ⍝ LAM IS THE CALL INITIATION RATE.
[7]  ⍝ MU IS THE CALL TERMINATION RATE.
[8]  ⍝ ALPHA IS THE TALKSPURT INITIATION RATE.
[9]  ⍝ BETA IS THE TALSPURT TERMINATION RATE.
[10] ⍝ A IS MAX NUMBER OF ACTIVE CALLS
[11] ⍝ C IS MAX NUMBER OF CALLS IN PROGRESS
[12] ⍝ COMPRESS IS THE COMPRESSION RATE
[13] ⍝ FOR PACKETIZED TALKSPURTS.
[14] ⍝ HEADER IS THE PROPORTION OF HEADER INFO
[15] ⍝ TO MEAN TALKSPURT LENGTH.
[16] ⍝ B IS THE INCOMING RATE IN BITS/SEC.
[17]  LAM←CD[1]
[18]  MU←CD[2]
[19]  ALPHA←CD[3]
[20]  BETA←CD[4]
[21]  A←CD[5]
[22]  C←CD[6]
[23]  COMPRESS←CD[7]
[24]  HEADER←CD[8]
[25]  B←32000
[26] ⍝ RO IS THE RATIO OF THE INPUT TO OUTPUT
[27] ⍝ TRANSMISSION RATES × B.
[28]  RO←A×B
[29]  SIZE←(+/⍳(A+1))
[30]  SIZE←SIZE+((C-A)×(A+1))
[31]  M←(SIZE,SIZE)⍴0
[32] ⍝ PROCESSOR SHARING SERVICE
[33] ⍝ BETAM IS THE TALKSPURT TERMINATION RATE AFTER
[34] ⍝ ACCOUNTING FOR COMPRESSION AND HEADER.
[35]  INVBETAM←((1÷COMPRESS)+HEADER)×B×INVBETA←1÷BETA
[36]  BETAM←1÷INVBETAM
[37]  SERV←BETAM×RO
```

```
[38]  M[1;1]←0
[39]  M[1;3]←LAM
[40]  CC←0
[41]  FINISH←1
[42] ITER:
[43]  START←FINISH+1
[44]  CC←CC+1
[45]  LEV←CC
[46]  →REACHA×⍳(LEV=A)
[47]  →LARGA×⍳(LEV>A)
[48]  NUMB←LEV+1
[49]  FINISH←START+(NUMB-1)
[50]  ML←((LEV+1),LEV)⍴0
[51]  MM←((LEV+1),(LEV+1))⍴0
[52]  MR←((LEV+1),(LEV+2))⍴0
[53]  →NEXTMA
[54] ⍝ NUMB OF CALLS IN PROGRESS = MAX NUMB OF ACTIVE CALLS
A
[55] REACHA:
[56]  NUMB←A+1
[57]  FINISH←START+(NUMB-1)
[58]  MR←MM←((A+1),(A+1))⍴0
[59]  ML←((A+1),A)⍴0
[60]  →NEXTMA
[61] ⍝ NUMB OF CALLS IN PROG > MAX NUMB OF ACTIVE CALLS
[62] LARGA:
[63]  NUMB←A+1
[64]  FINISH←START+(NUMB-1)
[65]  MR←MM←ML←((A+1),(A+1))⍴0
[66] NEXTMA:
[67]  CC1←0
[68] INNERR:
[69]  →OUTR×⍳(CC1=(LEV+1))
[70]  CC1←CC1+1
[71]  →INNERR1×⍳(CC1=(LEV+1))
[72]  ML[CC1;CC1]←(LEV-(CC1-1))×MU
[73] INNERR1:
[74]  →INNERR2×⍳(CC1=1)
[75]  ML[CC1;(CC1-1)]←(CC1-1)×MU
[76] INNERR2:
[77]  →INNERR×⍳(CC1<(⍴ML)[1])
[78] OUTR:
[79]  CM←0
[80] INNERM:
[81]  →OUTM×⍳(CM>A+1)
[82]  CM←CM+1
```

```
[83]   →NEXTM1×ι(CM=(ρMM)[1])
[84]   MM[CM;CM+1]←(LEV-(CM-1))×ALPHA
[85]  NEXTM1:
[86]   →NEXTM×ι(CM=1)
[87]   MM[CM;CM-1]←SERV×(CM-1)
[88]  NEXTM:
[89]   →INNERM×ι(CM<(ρMM)[1])
[90]  OUTM:
[91]   CL←0
[92]  INNERL:
[93]   →OUTL×ι(CL≥A)
[94]   CL←CL+1
[95]   MR[CL;(CL+1)]←LAM
[96]   →INNERL×ι(CL<(ρML)[1])
[97]  OUTL:
[98]   START1←START-1
[99]   →CEQA×ι(LEV=A)
[100]   →CBIGA×ι(LEV>A)
[101]   M[START1+(ιNUMB);(START1+(ιNUMB))]←MM
[102]  M[START1+(ιNUMB);(START-NUMB)+ι(NUMB-1)]←ML
[103]  →END×ι(CC=C)
[104]  M[START1+(ιNUMB);(START1+NUMB+(ιNUMB+1))]←MR
[105]  →ITER×ι(CC<C)
[106]CEQA:
[107]  M[START1+(ιNUMB);(START1+(ιNUMB))]←MM
[108]  M[START1+(ιNUMB);(START-NUMB)+ι(NUMB-1)]←ML
[109]  →END×ι(CC=C)
[110]  M[START1+(ιNUMB);(START1+NUMB+(ιNUMB))]←MR
[111]  →ITER×ι(CC<C)
[112]CBIGA:
[113]  M[START1+(ιNUMB);(START1+(ιNUMB))]←MM
[114]  M[START1+(ιNUMB);(START1-NUMB)+ιNUMB]←ML
[115]  →END×ι(CC=C)
[116]  M[START1+(ιNUMB);(START1+NUMB+(ιNUMB))]←MR
[117]  →ITER×ι(CC<C)
[118]END:
[119]  IDENT←(ιSIZE)∘.=(ιSIZE)
[120]  IDENT←IDENT×((SIZE,SIZE)ρ(+/M))
[121]  MI←M←M-IDENT
[122]  M[;1]←1
[123]  LHS←(1,SIZE)ρ(1,((SIZE-1)ρ0))
[124]  PIA←LHS+.×(⌹M)
[125]  MATRIX←⍉(3,(ρ,PIA))ρSC,SA,(,PIA)
       ∇
```

APPENDIX D

The APL program APPROX calculates the limiting probabilities for both the Erlang and multiplexer models using approximation techniques. It calls the routine STATES to help format the output. BLOCK is used to compute the inner, outer, and combined inner-outer blocking probabilities for both the approximation (APPROX) and the actual (MATRIXM) calculations for comparison.

```
      ∇APPROX CMM
[1]   ⎕IO←1
[2]  ⍝ THIS PROGRAM REQUIRES A VECTOR OF 8 ELEMENTS AS
INPUT.
[3]  ⍝ IT CALCULATES THE LIMITING PROBABILITIES FOR THE
MULTIPLEXER
[4]  ⍝ AND ERLANG MODELS USING MATRIX INVERSION.
[5]  ⍝ L IS LAMBDA, THE CALL INITIATION RATE.
[6]  ⍝ M IS MU, THE CALL TERMINATION RATE.
[7]  ⍝ ALPHA IS THE TALKSPURT INITIATION RATE.
[8]  ⍝ BETA IS THE TALKSPURT TERMINATION RATE.
[9]  ⍝ A IS THE NUMBER OF CHANNELS.
[10] ⍝ C IS THE MAXIMUM NUMBER OF CALLS ALLOWED.
[11] ⍝ COMPRESS IS THE COMPRESSION RATE OF PACKETIZED
TALKSPURTS.
[12] ⍝ HEADER IS THE PROPORTION OF HEADER INFORMATION TO
[13] ⍝ MEAN LENGTH OF TALKSPURT.
[14] ⍝ B IS THE INCOMING RATE IN BITS/SEC.
[15]  L←CMM[1]
[16]  M←CMM[2]
[17]  ALPHA←CMM[3]
[18]  BETA←CMM[4]
[19]  A←CMM[5]
[20]  C←CMM[6]
[21]  COMPRESS←CMM[7]
[22]  HEADER←CMM[8]
[23]  B←32000
[24]  RO←A×B
[25] ⍝ CALCULATION FOR BINOMIAL PROB. OF J TALKSPURTS GIVEN
K CHANNELS
[26] ⍝ FOR ERLANG
[27]  ⎕IO←0
[28] ⍝DIM←(A+1),(A+1)
[29] ⍝PRA←DIMρ0
[30] ⍝A1←ALPHA÷ALPHA+BETA
[31] ⍝A2←BETA÷ALPHA+BETA
```

```
[32] ⍝K←0
[33] ⍝J←⍳A+1
[34] ⍝INLP:PRA[K;J]←(J!K)×(A1*J)×(A2*(K-J))
[35] ⍝K←K+1
[36] ⍝→(K≤A)/INLP
[37] ⍝ FOR MUX
[38] ⍝ INVBETAM IS THE NEW TALKSPURT LENGTH IN BITS
[39] ⍝ AFTER COMPRESSION AND HEADER ARE CONSIDERED.
[38]  INVBETAM←((1÷COMPRESS)+HEADER)×B×INVBETA←1÷BETA
[39]  BETAM←(1÷INVBETAM)
[40]  SERV←BETAM×RO
[41]  DIMM←(C+1),(A+1)
[42]  PRAM←DIMM⍴0
[43]  K←0
[44]  J←⍳A+1
[45] INLPM:PRAM[K;J]←(J!K)×((ALPHA÷SERV)*J)
[46]  K←K+1
[47]  →(K≤C)/INLPM
[48]  PO←((A+1),(C+1))⍴1÷(+/PRAM)
[49]  PRAM←⍉PO×⍉PRAM
[50] ⍝ TRUNCATED POISSON PROBABILITY OF K CALLERS
[51] ⍝ GIVEN MAX J CHANNELS
[52] ⍝ FOR ERLANG (MAX CHANNELS = MAX CALLERS = A)
[53]  LOAD←L÷M
[54] ⍝K←⍳A+1
[55] ⍝PRC←(LOAD*K)÷!K
[56] ⍝PRCE←PRC÷(+/PRC)
[57] ⍝ALPE←PRA×⍉DIM⍴PRCE
[58] ⍝APOUTBLE←+/ALPE[A;]
[59] ⍝ FOR MUX (MAX CHANNELS = A, MAX CALLERS = C)
[60] ⍝K←(⍳C-A)+A+1
[61] ⍝PIO←PRC,(LOAD*K)÷!K
[62]  K←⍳C+1
[63]  PIO2←(C+1)⍴1
[64]  I←0
[65] MKPIO:X←1
[66] MKPIO2:PIO2[I]←(LOAD÷X)×PIO2[I]
[67]  X←X+1
[68]  →(X≤K[I])/MKPIO2
[69]  I←I+1
[70]  →(I≤C)/MKPIO
[71] ⍝PRCM←PIO÷+/PIO
[72]  PRCM2←PIO2÷+/PIO2
[73]  DIMM2←(A+1),(C+1)
[74] ⍝ALPM←PRAM×⍉DIMM2⍴PRCM
[75] ⍝ FORM MATRIX OUTPUT
```

```
[76] ⍝STATES(A,C)
[77] ⍝ALPE←(,ALPE>0)/,ALPE
[78] ⍝ALPM←(,ALPM>0)/,ALPM
[79] ⍝ALPE←ALPE,((⍴SA)-(⍴ALPE))⍴0⍝
[80] ⍝ALPM←ALPM,((⍴SA)-(⍴ALPM))⍴0
[81] ⍝MATRIXAP←⍉(4,(⍴SA))⍴SC,SA,(ALPE),(ALPM)
[82]  ⎕IO←1
     ∇

      ∇STATES CM
[1]  ⍝ THIS FUNCTION RETURNS 2 VECTORS WHICH,
[2]  ⍝ TOGETHER, GIVE THE STATES IN TERMS OF
[3]  ⍝ NUMBER OF CALLS AND ACTIVE CALLS (C,A)
[4]   ⎕IO←1
[5]   A←CM[1]
[6]   C←CM[2]
[7]   VV←0,⍳A
[8]   SA←⍳0
[9]   SC←⍳0
[10]  SA←SA,0
[11]  SC←SC,0
[12]  LEV←0
[13] ITERS:
[14]  LEV←LEV+1
[15]  SA←SA,(VV[⍳(LEV+1)])
[16]  SC←SC,((LEV+1)⍴LEV)
[17]  →ITERS×⍳(LEV<A)
[18]  →END×⍳(A=C)
[19] ITERB:
[20]  LEV←LEV+1
[21]  SA←SA,VV
[22]  SC←SC,((⍴VV)⍴LEV)
[23]  →ITERB×⍳(LEV<C)
[24] END:
     ∇
     ∇BLOCK CM
[1]   APPROX CM
[2]   MATRIXM CM
[3]   INBL←+/(SA=A)/,PIA
[4]   OUTBL←+/(SC=C)/,PIA
[5]   INOUTBL←INBL+OUTBL-¯1↑,PIA
[6]   INBLA←+/(SA=A)/ALPM
[7]   OUTBLA←+/(SC=C)/ALPM
[8]   INOUTBLA←INBLA+OUTBLA-¯1↑ALPM
[9]   MATRIX2←⍉(4,(⍴,PIA))⍴SC,SA,(ALPM),(,PIA)
     ∇
```

APPENDIX E

These tables compare actual outer and inner blocking probabilities (OUTBL, INBL) with their approximated counterparts (OUTBLA, INBLA). Results are shown for C (maximum number of calls allowed) equal to 5, 10, 15, 20, and 30. The values for A indicate the number of available channels.

The level of speech activity (average proportion of time a call of infinite duration is active) is assumed to be 35% for all runs. The mean length of a call is 3 minutes. The mean length of a talkspurt (β^{-1}) is 352 ms. The compression factor is 4-to-1 and the length of the header information is 15.625% of the mean length of a talkspurt. The rate of each active incoming channel is b = 32 Kbps. Thus, the value of $(\beta^{-1})^*$ is 4576 bits. The outgoing channel rate s is equal to A, the number of available channels, multiplied by b, the incoming channel rate. The values for load (λ/μ) are as indicated in the tables.

C • 5   A • 1

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.000600	.003067	.139541	.155796
2.0	.008525	.036697	.228909	.268848
3.0	.031201	.110054	.291287	.344872
4.0	.067800	.199067	.336466	.393533

C • 5   A • 2

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.002817	003067	.004368	.004396
2.0	.033933	.036697	.014415	.014638
3.0	.102803	.110054	.025236	.025672
4.0	.187823	.199067	.034265	.034802

C • 5   A • 3

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.003063	.003067	.000047	.000046
2.0	.036645	.036697	.000272	.000269
3.0	.109912	.110054	.000606	.000601
4.0	.198840	.199067	.000935	.000926

C • 5   A • 4

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.003067	.003067	.000000	.000000
2.0	.036697	036697	.000002	.000002
3.0	.110054	.110054	.000005	.000005
4.0	.199066	.199067	.000009	.000009
5.0	.284866	.284868	.000012	.000012

C • 5   A • 5

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.003067	.003067	.000000	.000000
2.0	.036697	.036697	.000000	.000000
3.0	.110054	.110054	.000000	.000000
4.0	.199067	.199067	.000000	.000000

C • 10   A • 1

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.000000	.000000	.139563	.156043
2.0	.000000	.000038	.229440	.274026
3.0	.000002	.000810	.293849	.365335
4.0	.000022	.005308	.343124	.437013
5.0	.000106	.018385	.382511	.493130
6.0	.000361	.043142	.414993	.536284
7.0	.000952	.078741	.442406	.568775
8.0	.002097	.121661	.465946	.592908
9.0	.004034	.167963	.486426	.610788

C • 10   A • 2

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.000000	.000000	.004417	.004454
2.0	.000018	.000038	.015691	.016187
3.0	.000390	.000810	.031280	.033097
4.0	.002651	.005308	.049188	.053231
5.0	.009672	.018385	.067782	.074441
6.0	.024096	.043142	.085720	.094647
7.0	.046763	.078741	.102057	.112434
8.0	.076569	.121661	.116316	.127276
9.0	.111361	.167963	.128415	.139294

C • 10   A • 3

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	000000	.000000	.000050	.000050
2.0	.000036	.000038	.000373	.000371
3.0	.000773	.000810	.001165	.001161
4.0	.005068	.005308	.002505	.002503
5 0	.017601	.018385	.004306	.004312
6.0	.041446	.043142	.006346	.006362
7.0	.075951	.078741	.008387	.008408
8.0	.117754	.121661	.010262	.010282
9.0	.163122	.167963	.011900	.011913

C • 10   A • 4

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.000000	.000000	.000000	.000000
2.0	.000038	.000038	.000004	.000004
3.0	.000809	.000810	.000021	.000021
4.0	.005300	.005308	.000059	.000058
5.0	.018359	.018385	.000122	.000120
6.0	.043085	.043142	.000203	.000200
7.0	.078645	.078741	.000291	.000288
8 0	.121526	.121661	.000378	.000374
9.0	.167793	.167963	.000458	000453

C • 10   A • 5

LOAD	OUTBL	OUTBLA	INBL	INBLA
1.0	.000000	.000000	.000000	.000000
2.0	.000038	.000038	.000000	.000000
3.0	.000810	.000810	.000000	.000000
4.0	.005307	.005308	.000001	.000001
5.0	.018384	.018385	.000002	.000002
6.0	.043141	.043142	.000003	.000003
7.0	.078739	.078741	.000005	.000005
8.0	.121659	.121661	.000007	.000007
9.0	.167960	.167963	.000009	.000009

C • 15   A • 1

LOAD	OUTBL	DUTBLA	INBL	INBLA
5.0	.000000	.000157	.382516	.496189
6.0	.000000	.000892	.415014	.544033
7.0	.000000	.003319	.442467	.583626
8.0	.000000	.009101	.466091	.616453
9.0	.000001	.019868	.486723	.643499
10.0	.000003	.036497	.504960	.665527
11.0	.000008	.058797	.521245	.683244
12.0	.000018	.085729	.535910	.697359
13.0	.000037	.115865	.549215	.708549

C • 15   A • 2

LOAD	OUTBL	OUTBLA	INBL	INBLA
5.0	.000019	.000157	.068656	.076777
6.0	.000116	.000892	.088285	.101180
7.0	.000475	.003319	.107725	.126049
8.0	.001456	.009101	.126616	.150441
9.0	.003596	.019868	.144696	.173431
10.0	.007514	.036497	.161762	.194273
11.0	.013778	.058797	.177653	.212531
12.0	.022766	.085729	.192256	.228107
13.0	.034601	.115865	.205512	.241158

C • 15   A • 3

LOAD	OUTBL	OUTBLA	INBL	INBLA
5.0	.000124	.000157	.004685	.004725
6.0	.000707	.000892	.007487	.007605
7.0	.002654	.003319	.010934	.011183
8.0	.007362	.009101	.014887	.015308
9.0	.016292	.019868	.019132	.019737
10.0	.030580	.036497	.023430	.024196
11.0	.049703	.058797	.027567	.028446
12.0	.073562	.085729	.031592	.032330
13.0	.100820	.115865	.034826	.035773

C • 15   A • 4

LOAD	OUTBL	OUTBLA	INBL	INBLA
5.0	.000155	.000157	.000149	.000148
6.0	.000879	.000892	.000291	.000288
7.0	.003273	.003319	.000499	.000495
8.0	.008981	.009101	.000775	.000767
9.0	.019621	.019868	.001103	.001093
10.0	.036070	.036497	.001462	.001449
11.0	.058159	.058797	.001828	.001812
12.0	.084872	.085729	.002182	.002163
13.0	.114800	.115865	.002509	.002487

```
C = 15   A = 5
LOAD     OUTBL     OUTBLA    INBL      INBLA
  5.0   .000157   .000157   .000003   .000003
  6.0   .000891   .000892   .000007   .000006
  7.0   .003317   .003319   .000013   .000013
  8.0   .009097   .009101   .000023   .000022
  9.0   .019860   .019868   .000035   .000034
 10.0   .036482   .036497   .000049   .000049
 11.0   .058774   .058797   .000065   .000064
 12.0   .085698   .085729   .000081   .000079
 13.0   .115826   .115865   .000095   .000094
____________________________________________
C = 20   A = 1
LOAD     OUTBL     OUTBLA    INBL      INBLA
 10.0   .000000   .001869   .504960   .671318
 11.0   .000000   .004640   .521265   .692619
 12.0   .000000   .009796   .535911   .710936
 13.0   .000000   .018110   .549217   .726588
 14.0   .000000   .030035   .561365   .739840
 15.0   .000000   .045593   .572518   .750952
 16.0   .000000   .064411   .582809   .760194
 17.0   .000000   .085860   .592346   .767834
 18.0   .000000   .109213   .601218   .774134

C = 20   A = 2
LOAD     OUTBL     OUTBLA    INBL      INBLA
 10.0   .000041   .001869   .162731   .202251
 11.0   .000122   .004640   .179552   .226056
 12.0   .000307   .009796   .195578   .248492
 13.0   .000687   .018110   .210814   .269187
 14.0   .001383   .030035   .225270   .287860
 15.0   .002552   .045593   .238957   .304366
 16.0   .004371   .064411   .251880   .318709
 17.0   .007017   .085860   .264043   .331011
 18.0   .010655   .109213   .275449   .341672

C = 20   A = 3
LOAD     OUTBL     OUTBLA    INBL      INBLA
 10.0   .001019   .001869   .025455   .026948
 11.0   .002598   .004640   .031223   .033336
 12.0   .005655   .009796   .037253   .040022
 13.0   .010813   .018110   .043376   .046764
 14.0   .018579   .030035   .049423   .053321
 15.0   .029229   .045593   .055238   .059497
 16.0   .042758   .064411   .060703   .065160
 17.0   .058914   .085860   .065737   .070243
 18.0   .077278   .109213   .070304   .074738

C = 20   A = 4
LOAD     OUTBL     OUTBLA    INBL      INBLA
 10.0   .001767   .001869   .001807   .001800
 11.0   .004394   .004640   .002464   .002458
 12.0   .009299   .009796   .003216   .003212
 13.0   .017239   .018110   .004036   .004034
 14.0   .028677   .030035   .004888   .004887
 15.0   .043667   .045593   .005736   .005735
 16.0   .061882   .064411   .006552   .006549
 17.0   .082738   .085860   .007316   .007309
 18.0   .105538   .109213   .008017   .008004

C = 20   A = 5
LOAD     OUTBL     OUTBLA    INBL      INBLA
 10.0   .001863   .001869   .000071   .000070
 11.0   .004626   .004640   .000106   .000105
 12.0   .009768   .009796   .000150   .000148
 13.0   .018061   .018110   .000201   .000198
 14.0   .029958   .030035   .000256   .000252
 15.0   .045483   .045593   .000313   .000308
 16.0   .064265   .064411   .000370   .000365
 17.0   .085678   .085860   .000425   .000419
 18.0   .108997   .109213   .000476   .000470
```

```
C = 20   A = 6
LOAD     OUTBL     OUTBLA    INBL      INBLA
 10.0   .001869   .001869   .000002   .000002
 11.0   .004640   .004640   .000003   .000003
 12.0   .009795   .009796   .000004   .000004
 13.0   .018108   .018110   .000006   .000006
 14.0   .030033   .030035   .000008   .000008
 15.0   .045589   .045593   .000011   .000010
 16.0   .064406   .064411   .000013   .000013
 17.0   .085854   .085860   .000015   .000015
 18.0   .109205   .109213   .000017   .000017
____________________________________________
C = 30   A = 1
LOAD     OUTBL     OUTBLA    INBL      INBLA
 10.0   .000000   .000000   .504960   .671565
 20.0   .000000   .008457   .617259   .807340
 30.0   .000000   .132460   .674998   .849465

C = 30   A = 2
LOAD     OUTBL     OUTBLA    INBL      INBLA
 15.0   .000000   .000221   .239302   .316978
 16.0   .000000   .000564   .252495   .337216
 17.0   .000000   .001281   .265068   .356489
 18.0   .000000   .002622   .277059   .374743
 19.0   .000001   .004902   .288505   .391912
 20.0   .000003   .008457   .299441   .407918
 21.0   .000005   .013594   .309900   .422690
 22.0   .000011   .020535   .319911   .436175
 23.0   .000022   .029386   .329505   .448352
 24.0   .000040   .040121   .338705   .459238
 25.0   .000070   .052603   .347537   .468885
 26.0   .000119   .066612   .356022   .477375
 27.0   .000194   .081880   .364180   .484809
 28.0   .000307   .098122   .372028   .491296
 29.0   .000471   .115065   .379582   .496948

C = 30   A = 3
LOAD     OUTBL     OUTBLA    INBL      INBLA
 15.0   .000024   .000221   .058605   .066333
 16.0   .000066   .000564   .065968   .075567
 17.0   .000161   .001281   .073451   .085073
 18.0   .000354   .002622   .081003   .094737
 19.0   .000719   .004902   .088577   .104422
 20.0   .001352   .008457   .096125   .113980
 21.0   .002379   .013594   .103599   .123255
 22.0   .003942   .020535   .110949   .132106
 23.0   .006193   .029386   .118123   .140417
 24.0   .009278   .040121   .125073   .148106
 25.0   .013321   .052603   .131753   .155129
 26.0   .018412   .066612   .138123   .161476
 27.0   .024603   .081880   .144154   .167165
 28.0   .031901   .098122   .149823   .172234
 29.0   .040272   .115065   .155118   .176733

C = 30   A = 4
LOAD     OUTBL     OUTBLA    INBL      INBLA
 15.0   .000157   .000221   .007023   .007137
 16.0   .000404   .000564   .008581   .008760
 17.0   .000926   .001281   .010306   .010568
 18.0   .001916   .002622   .012184   .012543
 19.0   .003626   .004902   .014190   .014657
 20.0   .006340   .008457   .016291   .016869
 21.0   .010336   .013594   .018447   .019130
 22.0   .015848   .020535   .020616   .021390
 23.0   .023025   .029386   .022755   .023600
 24.0   .031915   .040121   .024828   .025719
 25.0   .042465   .052603   .026804   .027718
 26.0   .054539   .066612   .028661   .029577
 27.0   .067940   .081880   .030388   .031285
 28.0   .082440   .098122   .031979   .032842
 29.0   .097799   .115065   .033433   .034252
```

```
C = 30   A = 5
LOAD      OUTBL     OUTBLA     INBL      INBLA
  15.0   .000214   .000221   .000451   .000446
  16.0   .000547   .000564   .000594   .000588
  17.0   .001241   .001281   .000766   .000758
  18.0   .002544   .002622   .000965   .000955
  19.0   .004760   .004902   .001191   .001180
  20.0   .008222   .008457   .001439   .001426
  21.0   .013231   .013594   .001705   .001690
  22.0   .020013   .020535   .001981   .001964
  23.0   .028677   .029386   .002260   .002242
  24.0   .039206   .040121   .002537   .002516
  25.0   .051474   .052603   .002805   .002782
  26.0   .065268   .066612   .003061   .003035
  27.0   .080329   .081880   .003302   .003273
  28.0   .096377   .098122   .003525   .003493
  29.0   .113144   .115065   .003722   .003697
```

```
C = 30   A = 6
LOAD      OUTBL     OUTBLA     INBL      INBLA
  15.0   .000221   .000221   .000018   .000018
  16.0   .000563   .000564   .000026   .000026
  17.0   .001278   .001281   .000036   .000035
  18.0   .002617   .002622   .000048   .000047
  19.0   .004893   .004902   .000062   .000061
  20.0   .008443   .008457   .000078   .000077
  21.0   .013571   .013594   .000096   .000094
  22.0   .020502   .020535   .000115   .000113
  23.0   .029340   .029386   .000135   .000133
  24.0   .040062   .040121   .000156   .000153
  25.0   .052530   .052603   .000176   .000173
  26.0   .066525   .066612   .000196   .000192
  27.0   .081778   .081880   .000214   .000211
  28.0   .098007   .098122   .000232   .000228
  29.0   .114938   .115065   .000249   .000244
```

```
C = 30   A = 7
LOAD      OUTBL     OUTBLA     INBL      INBLA
  15.0   .000221   .000221   .000001   .000001
  16.0   .000564   .000564   .000001   .000001
  17.0   .001281   .001281   .000001   .000001
  18.0   .002622   .002622   .000002   .000002
  19.0   .004902   .004902   .000002   .000002
  20.0   .008457   .008457   .000003   .000003
  21.0   .013593   .013594   .000004   .000004
  22.0   .020534   .020535   .000005   .000004
  23.0   .029384   .029386   .000005   .000005
  24.0   .040118   .040121   .000006   .000006
  25.0   .052600   .052603   .000007   .000007
  26.0   .066609   .066612   .000008   .000008
  27.0   .081875   .081880   .000009   .000009
  28.0   .098117   .098122   .000010   .000010
  29.0   .115060   .115065   .000011   .000011
```

APPENDIX F

The following tables show results of the approximation to the multiplexer model when C (maximum number of calls allowed) equals 5, 10, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 125, 150, and 160. The values for A indicate the number of available channels.

Outer blocking probabilities are calculated from the formula for the truncated Poisson distribution (2). Inner blocking probabilities (equal to $P_J(k)$, from (6), where J = A) are compared for three different levels (28%, 35%, and 42%) of speech activity (average proportion of time a call of infinite duration is active). Mean talkspurt lengths (β^{-1}) are assumed to be 288 ms, 352 ms, and 420 ms, respectively, for the three levels of speech activity. The compression factor is 4-to-1 and the length of the header information is 15.625% of the mean length of a talkspurt. The rate of each active incoming channel is b = 32 Kbps. Thus, the values for $(\beta^{-1})^*$ are 3744, 4576, and 5460 bits, respectively, for the three levels of speech activity.

The mean length of a call is 3 minutes. The outgoing channel rate s is equal to A, the number of available channels, multiplied by b, the channel rate. The values for load (λ/μ) are as indicated in the tables.

```
-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 5   A = 1
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .003067   .121505   .155794   .190727
   2.0   .032258   .188530   .236326   .283025
   3.0   .099265   .251962   .311061   .367079
   4.0   .186047   .300867   .367794   .429927

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 5   A = 2
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .003067   .002460   .004396   .007169
   2.0   .032258   .007323   .012868   .020597
   3.0   .099265   .013315   .023156   .036636
   4.0   .186047   .018826   .032526   .051083

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 5   A = 3
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .003067   .000018   .000046   .000102
   2.0   .032258   .000095   .000237   .000515
   3.0   .099265   .000219   .000542   .001173
   4.0   .186047   .000351   .000866   .001868

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 5   A = 4
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .003067   .000000   .000000   .000001
   2.0   .032258   .000000   .000002   .000005
   3.0   .099265   .000001   .000005   .000014
   4.0   .186047   .000002   .000008   .000024

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 5   A = 5
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .003067   .000000   .000000   .000000
   2.0   .032258   .000000   .000000   .000000
   3.0   .099265   .000000   .000000   .000000
   4.0   .186047   .000000   .000000   .000000
-----------------------------------------------------
-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 10   A = 3
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .000000   .000020   .000050   .000109
   2.0   .000034   .000133   .000327   .000700
   3.0   .000737   .000438   .001056   .002217
   4.0   .005030   .001000   .002372   .004896
   5.0   .017896   .001791   .004197   .008540
   6.0   .042590   .002706   .006281   .012637
   7.0   .078266   .003627   .008357   .016671
   8.0   .121312   .004475   .010253   .020319

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 10   A = 4
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .000000   .000000   .000000   .000001
   2.0   .000034   .000001   .000004   .000011
   3.0   .000737   .000006   .000019   .000054
   4.0   .005030   .000016   .000055   .000156
   5.0   .017896   .000035   .000117   .000326
   6.0   .042590   .000060   .000197   .000546
   7.0   .078266   .000087   .000286   .000787
   8.0   .121312   .000113   .000373   .001022

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 10   A = 5
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .000000   .000000   .000000   .000000
   2.0   .000034   .000000   .000000   .000000
   3.0   .000737   .000000   .000000   .000001
   4.0   .005030   .000000   .000001   .000003
   5.0   .017896   .000000   .000002   .000007
   6.0   .042590   .000001   .000003   .000013
   7.0   .078266   .000001   .000005   .000020
   8.0   .121312   .000002   .000007   .000027

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 10   A = 6
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   1.0   .000000   .000000   .000000   .000000
   2.0   .000034   .000000   .000000   .000000
   3.0   .000737   .000000   .000000   .000000
   4.0   .005030   .000000   .000000   .000000
   5.0   .017896   .000000   .000000   .000000
   6.0   .042590   .000000   .000000   .000000
   7.0   .078266   .000000   .000000   .000000
   8.0   .121312   .000000   .000000   .000000
-----------------------------------------------------
-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 20   A = 3
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   5.0   .000000   .002036   .004729   .009533
   6.0   .000004   .003348   .007640   .015101
   7.0   .000030   .005059   .011344   .021995
   8.0   .000159   .007182   .015833   .030130
   9.0   .000617   .009715   .021064   .039367
  10.0   .001869   .012627   .026948   .049504
  11.0   .004640   .015855   .033336   .060265
  12.0   .009796   .019295   .040022   .071301
  13.0   .018110   .022822   .046764   .082230
  14.0   .030035   .026301   .053321   .092694
  15.0   .045593   .029618   .059497   .102417

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 20   A = 4
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
   5.0   .000000   .000045   .000148   .000407
   6.0   .000004   .000090   .000292   .000789
   7.0   .000030   .000160   .000513   .001364
   8.0   .000159   .000263   .000829   .002171
   9.0   .000617   .000403   .001256   .003238
  10.0   .001869   .000585   .001800   .004573
  11.0   .004640   .000808   .002458   .006161
  12.0   .009796   .001067   .003212   .007956
  13.0   .018110   .001353   .004034   .009885
  14.0   .030035   .001652   .004887   .011864
  15.0   .045593   .001952   .005735   .013813

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 20   A = 5
-----------------------------------------------------
LOAD      OUTER   INNER(28)  INNER(35)  INNER(42)
```

5.0	.000000	.000001	.000003	.000010
6.0	.000004	.000001	.000007	.000024
7.0	.000030	.000003	.000014	.000050
8.0	.000159	.000006	.000026	.000092
9.0	.000616	.000010	.000044	.000156
10.0	.001868	.000016	.000070	.000245
11.0	.004639	.000024	.000105	.000363
12.0	.009795	.000034	.000148	.000507
13.0	.018109	.000046	.000199	.000673
14.0	.030035	.000059	.000252	.000852
15.0	.045593	.000073	.000308	.001037

BLOCKING PROBABILITIES FOR C = 20 A = 6

LOAD	OUTER	INNER(29)	INNER(35)	INNER(42)
5.0	.000000	.000000	.000000	.000000
6.0	.000004	.000000	.000000	.000001
7.0	.000030	.000000	.000000	.000001
8.0	.000159	.000000	.000001	.000003
9.0	.000616	.000000	.000001	.000005
10.0	.001868	.000000	.000002	.000009
11.0	.004639	.000000	.000003	.000014
12.0	.009795	.000001	.000004	.000020
13.0	.018109	.000001	.000006	.000029
14.0	.030035	.000001	.000008	.000038
15.0	.045593	.000002	.000010	.000048

BLOCKING PROBABILITIES FOR C = 20 A = 7

LOAD	OUTER	INNER(28)	INNER(35)	INNER(42)
5.0	.000000	.000000	.000000	.000000
6.0	.000004	.000000	.000000	.000000
7.0	.000030	.000000	.000000	.000000
8.0	.000159	.000000	.000000	.000000
9.0	.000616	.000000	.000000	.000000
10.0	.001868	.000000	.000000	.000000
11.0	.004639	.000000	.000000	.000000
12.0	.009795	.000000	.000000	.000001
13.0	.018109	.000000	.000000	.000001
14.0	.030035	.000000	.000000	.000001
15.0	.045593	.000000	.000000	.000001

BLOCKING PROBABILITIES FOR C = 25 A = 4

LOAD	OUTER	INNER(28)	INNER(35)	INNER(42)
10.0	.000029	.000595	.001827	.004633
11.0	.000117	.000838	.002539	.006336
12.0	.000378	.001141	.003408	.008375
13.0	.001029	.001505	.004439	.010748
14.0	.002419	.001932	.005626	.013431
15.0	.005011	.002415	.006951	.016375
16.0	.009319	.002943	.008392	.019508
17.0	.015801	.003502	.009879	.022738
18.0	.024756	.004074	.011394	.025970
19.0	.036273	.004641	.012884	.029115
20.0	.050222	.005190	.014313	.032102

BLOCKING PROBABILITIES FOR C = 25 A = 5

LOAD	OUTER	INNER(28)	INNER(35)	INNER(42)
10.0	.000029	.000017	.000073	.000253
11.0	.000117	.000026	.000112	.000385
12.0	.000378	.000039	.000165	.000561
13.0	.001029	.000055	.000234	.000785
14.0	.002419	.000076	.000320	.001062
15.0	.005011	.000102	.000423	.001389
16.0	.009319	.000131	.000542	.001760
17.0	.015801	.000164	.000672	.002165
18.0	.024756	.000199	.000809	.002589
19.0	.036273	.000234	.000950	.003019
20.0	.050222	.000270	.001089	.003443

BLOCKING PROBABILITIES FOR C = 25 A = 6

LOAD	OUTER	INNER(28)	INNER(35)	INNER(42)
10.0	.000029	.000000	.000002	.000009
11.0	.000117	.000001	.000003	.000015
12.0	.000378	.000001	.000005	.000024
13.0	.001029	.000001	.000008	.000037
14.0	.002419	.000002	.000012	.000054
15.0	.005011	.000003	.000017	.000075
16.0	.009319	.000004	.000022	.000100
17.0	.015801	.000005	.000029	.000129
18.0	.024756	.000006	.000036	.000161
19.0	.036273	.000008	.000044	.000194
20.0	.050222	.000009	.000052	.000227

BLOCKING PROBABILITIES FOR C = 25 A = 7

LOAD	OUTER	INNER(28)	INNER(35)	INNER(42)
10.0	.000029	.000000	.000000	.000000
11.0	.000117	.000000	.000000	.000000
12.0	.000378	.000000	.000000	.000001
13.0	.001029	.000000	.000000	.000001
14.0	.002419	.000000	.000000	.000002
15.0	.005011	.000000	.000000	.000003
16.0	.009319	.000000	.000001	.000004
17.0	.015801	.000000	.000001	.000005
18.0	.024756	.000000	.000001	.000007
19.0	.036273	.000000	.000001	.000008
20.0	.050222	.000000	.000002	.000010

BLOCKING PROBABILITIES FOR C = 25 A = 8

LOAD	OUTER	INNER(28)	INNER(35)	INNER(42)
10.0	.000029	.000000	.000000	.000000
11.0	.000117	.000000	.000000	.000000
12.0	.000378	.000000	.000000	.000000
13.0	.001029	.000000	.000000	.000000
14.0	.002419	.000000	.000000	.000000
15.0	.005011	.000000	.000000	.000000
16.0	.009319	.000000	.000000	.000000
17.0	.015801	.000000	.000000	.000000
18.0	.024756	.000000	.000000	.000000
19.0	.036273	.000000	.000000	.000000
20.0	.050222	.000000	.000000	.000000

BLOCKING PROBABILITIES FOR C = 30 A = 4

LOAD	OUTER	INNER(28)	INNER(35)	INNER(42)
15.0	.000221	.002490	.007137	.016743
16.0	.000564	.003097	.008760	.020247
17.0	.001281	.003784	.010568	.024080
18.0	.002622	.004547	.012543	.028193
19.0	.004902	.005375	.014657	.032522
20.0	.008457	.006253	.016869	.036983
21.0	.013594	.007162	.019130	.041481

```
  22.0   .020535   .008079   .021390   .045920
  23.0   .029386   .008986   .023600   .050214
  24.0   .040121   .009863   .025719   .054290
  25.0   .052603   .010696   .027718   .058099

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 30   A = 5
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  15.0   .000221   .000108   .000446   .001454
  16.0   .000564   .000144   .000588   .001893
  17.0   .001281   .000187   .000758   .002410
  18.0   .002622   .000238   .000955   .003005
  19.0   .004902   .000297   .001180   .003670
  20.0   .008457   .000362   .001426   .004394
  21.0   .013594   .000432   .001690   .005159
  22.0   .020535   .000505   .001964   .005948
  23.0   .029386   .000580   .002242   .006740
  24.0   .040121   .000655   .002516   .007517
  25.0   .052603   .000727   .002782   .008264

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 30   A = 6
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  15.0   .000221   .000003   .000018   .000081
  16.0   .000564   .000004   .000026   .000114
  17.0   .001281   .000006   .000035   .000154
  18.0   .002622   .000008   .000047   .000204
  19.0   .004902   .000011   .000061   .000262
  20.0   .008457   .000013   .000077   .000328
  21.0   .013594   .000017   .000094   .000402
  22.0   .020535   .000020   .000113   .000480
  23.0   .029386   .000024   .000133   .000561
  24.0   .040121   .000027   .000153   .000642
  25.0   .052603   .000031   .000173   .000723

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 30   A = 7
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  15.0   .000221   .000000   .000001   .000003
  16.0   .000564   .000000   .000001   .000005
  17.0   .001281   .000000   .000001   .000007
  18.0   .002622   .000000   .000002   .000009
  19.0   .004902   .000000   .000002   .000013
  20.0   .008457   .000000   .000003   .000017
  21.0   .013594   .000000   .000004   .000021
  22.0   .020535   .000001   .000004   .000026
  23.0   .029386   .000001   .000005   .000031
  24.0   .040121   .000001   .000006   .000037
  25.0   .052603   .000001   .000007   .000042

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 30   A = 8
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  15.0   .000221   .000000   .000000   .000000
  16.0   .000564   .000000   .000000   .000000
  17.0   .001281   .000000   .000000   .000000
  18.0   .002622   .000000   .000000   .000000
  19.0   .004902   .000000   .000000   .000000
  20.0   .008457   .000000   .000000   .000001
  21.0   .013594   .000000   .000000   .000001
  22.0   .020535   .000000   .000000   .000001
  23.0   .029386   .000000   .000000   .000001
  24.0   .040121   .000000   .000000   .000001
  25.0   .052603   .000000   .000000   .000002

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 30   A = 9
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  15.0   .000221   .000000   .000000   .000000
  16.0   .000564   .000000   .000000   .000000
  17.0   .001281   .000000   .000000   .000000
  18.0   .002622   .000000   .000000   .000000
  19.0   .004902   .000000   .000000   .000000
  20.0   .008457   .000000   .000000   .000000
  21.0   .013594   .000000   .000000   .000000
  22.0   .020535   .000000   .000000   .000000
  23.0   .029386   .000000   .000000   .000000
  24.0   .040121   .000000   .000000   .000000
  25.0   .052603   .000000   .000000   .000000
--------------------------------------------------

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 35   A = 4
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  20.0   .000686   .006502   .017432   .037996
  21.0   .001393   .007584   .020082   .043181
  22.0   .002616   .008748   .022889   .048580
  23.0   .004578   .009981   .025818   .054123
  24.0   .007514   .011263   .028823   .059726
  25.0   .011646   .012572   .031854   .065301
  26.0   .017149   .013884   .034859   .070759
  27.0   .024128   .015177   .037789   .076020
  28.0   .032606   .016429   .040601   .081018
  29.0   .042527   .017624   .043263   .085704
  30.0   .053771   .018751   .045752   .090049

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 35   A = 5
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  20.0   .000686   .000388   .001517   .004633
  21.0   .001393   .000477   .001845   .005570
  22.0   .002616   .000577   .002213   .006602
  23.0   .004578   .000689   .002616   .007719
  24.0   .007514   .000810   .003048   .008906
  25.0   .011646   .000938   .003501   .010133
  26.0   .017149   .001070   .003967   .011384
  27.0   .024128   .001205   .004437   .012632
  28.0   .032606   .001338   .004900   .013855
  29.0   .042527   .001469   .005350   .015033
  30.0   .053771   .001595   .005780   .016153

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 35   A = 6
--------------------------------------------------
LOAD      OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  20.0   .000686   .000015   .000085   .000360
  21.0   .001393   .000019   .000108   .000456
  22.0   .002616   .000025   .000136   .000568
  23.0   .004578   .000031   .000168   .000694
  24.0   .007514   .000037   .000203   .000834
  25.0   .011646   .000045   .000242   .000984
  26.0   .017149   .000052   .000283   .001142
  27.0   .024128   .000060   .000325   .001305
  28.0   .032606   .000069   .000367   .001468
  29.0   .042527   .000077   .000410   .001629
  30.0   .053771   .000085   .000451   .001785

--------------------------------------------------
BLOCKING PROBABILITIES FOR C. = 35   A = 7
--------------------------------------------------
```

```
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  20.0    .000686   .000000   .000003   .000019
  21.0    .001393   .000001   .000006   .000026
  22.0    .002616   .000001   .000006   .000033
  23.0    .004578   .000001   .000007   .000043
  24.0    .007514   .000001   .000009   .000053
  25.0    .011646   .000001   .000011   .000065
  26.0    .017149   .000002   .000014   .000077
  27.0    .024128   .000002   .000016   .000090
  28.0    .032606   .000002   .000019   .000104
  29.0    .042527   .000003   .000021   .000117
  30.0    .053771   .000003   .000024   .000131

----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 35   A = 8
----------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  20.0    .000686   .000000   .000000   .000001
  21.0    .001393   .000000   .000000   .000001
  22.0    .002616   .000000    000000   .000001
  23.0    .004578   .000000    000000   .000002
  24.0    .007514   .000000   .000000   .000002
  25.0    .011646   .000000   .000000   .000003
  26.0    .017149   .000000   .000000   .000004
  27.0    .024128   .000000   .000001   .000004
  28.0    .032606   .000000   .000001   .000005
  29.0    .042527   .000000   .000001   .000006
  30.0    .053771   .000000   .000001   .000007

----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 35   A = 9
----------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  15.0    .000004   .000000   .000000   .000000
  16.0    .000015   .000000   .000000   .000000
  17.0    .000047   .000000   .000000   .000000
  18.0    .000127   .000000   .000000   .000000
  19.0    .000310   .000000   .000000   .000000
  20.0    .000686   .000000   .000000   .000000
  21.0    .001393   .000000   .000000   .000000
  22.0    .002616   .000000   .000000   .000000
  23.0    .004578   .000000   .000000   .000000
  24.0    .007514   .000000   .000000   .000000
  25.0    .011646   .000000   .000000   .000000
  26.0    .017149   .000000   .000000   .000000
  27.0    .024128   .000000   .000000   .000000
  28.0    .032606   .000000   .000000   .000000
  29.0    .042527   .000000   .000000   .000000
  30.0    .053771   .000000   .000000   .000000
----------------------------------------------------
----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 40   A = 4
----------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  25.0    .001411   .013125   .033008   .067203
  26.0    .002497   .014726   .036606   .073619
  27.0    .004170   .016393   .040298   .080102
  28.0    .006605   .018135   .044039   .086578
  29.0    .009971   .019838   .047780   .092967
  30.0    .014409   .021567   .051470   .099192
  31.0    .020017   .023268   .055061   .105183
  32.0    .026838   .024917   .058512   .110879
  33.0    .034864   .026497   .061788   .116238
  34.0    .044032   .027993   .064866   .121229
  35.0    .054244   .029397   .067731   .125840

----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 40   A = 5
```

```
----------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  25.0    .001411   .001010   .003738   .010713
  26.0    .002497   .001183   .004331   .012271
  27.0    .004170   .001369   .004967   .013920
  28.0    .006605   .001568   .005638   .015637
  29.0    .009971   .001776   .006333   .017398
  30.0    .014409   .001990   .007042   .019173
  31.0    .020017   .002206   .007752   .020937
  32.0    .026838   .002421   .008454   .022663
  33.0    .034864   .002632   .009136   .024329
  34.0    .044032   .002836   .009792   .025916
  35.0    .054244   .003030   .010414   .027414

----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 40   A = 6
----------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  20.0    .000028   .000015   .000086   .000364
  21.0    .000072   .000020   .000111   .000465
  22.0    .000170   .000026   .000141   .000586
  23.0    .000371   .000032   .000177   .000728
  24.0    .000748   .000040   .000220   .000893
  25.0    .001411   .000050   .000269   .001081
  26.0    .002497   .000061   .000324   .001292
  27.0    .004170   .000073   .000386   .001525
  28.0    .006605   .000086   .000454   .001777
  29.0    .009971   .000101   .000526   .002044
  30.0    .014409   .000116   .000602   .002321
  31.0    .020017   .000132   .000680   .002605
  32.0    .026838   .000148   .000759   .002889
  33.0    .034864   .000164   .000837   .003169
  34.0    .044032   .000179   .000913   .003442
  35.0     054244   .000194   .000987   .003705

----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 40   A = 7
----------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  20.0    .000028   .000000   .000003   .000020
  21.0    .000072   .000001   .000005   .000027
  22.0    .000170   .000001   .000006   .000035
  23.0    .000371   .000001   .000008   .000046
  24.0    .000748   .000001   .000010   .000059
  25.0    .001411   .000002   .000013   .000074
  26.0    .002497   .000002   .000017   .000093
  27.0    .004170   .000003   .000021   .000113
  28.0    .006605   .000003   .000025   .000136
  29.0    .009971   .000004   .000030   .000162
  30.0    .014409   .000005   .000035   .000189
  31.0    .020017   .000005   .000040   .000217
  32.0    .026838   .000006   .000046   .000245
  33.0    .034864   .000007   .000052   .000274
  34.0    .044032   .000008   .000057   .000303
  35.0    .054244   .000008   .000063   .000331

----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 40   A = 8
----------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
----------------------------------------------------
  20.0    .000028   .000000   .000000   .000001
  21.0    .000072   .000000   .000000   .000001
  22.0    .000170   .000000   .000000   .000002
  23.0    .000371   .000000   .000000   .000002
  24.0    .000748   .000000   .000000   .000003
  25.0    .001411   .000000   .000000   .000004
  26.0    .002497   .000000   .000001   .000005
  27.0    .004170   .000000   .000001   .000006
  28.0   .006605   .000000   .000001   .000008
  29.0   .009971   .000000   .000001   .000009
  30.0   .014409   .000000   .000001   .000011
  31.0   .020017   .000000   .000002   .000013
  32.0   .026838   .000000   .000002   .000015
  33.0   .034864   .000000   .000002   .000017
  34.0   .044032   .000000   .000003   .000019
  35.0   .054244   .000000   .000003   .000021

------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 40    A = 9
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  20.0   .000028   .000000   .000000   .000000
  21.0   .000072   .000000   .000000   .000000
  22.0   .000170   .000000   .000000   .000000
  23.0   .000371   .000000   .000000   .000000
  24.0   .000748   .000000   .000000   .000000
  25.0   .001411   .000000   .000000   .000000
  26.0   .002497   .000000   .000000   .000000
  27.0   .004170   .000000   .000000   .000000
  28.0   .006605   .000000   .000000   .000000
  29.0   .009971   .000000   .000000   .000000
  30.0   .014409   .000000   .000000   .000000
  31.0   .020017   .000000   .000000   .000001
  32.0   .026838   .000000   .000000   .000001
  33.0   .034864   .000000   .000000   .000001
  34.0   .044032   .000000   .000000   .000001
  35.0   .054244   .000000   .000000   .000001
------------------------------------------------------
------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 45    A = 4
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  28.0   .000767   .018550   .044868   .087856
  29.0   .001369   .020499   .049061   .094929
  30.0   .002320   .022545   .053353   .102059
  31.0   .003744   .024649   .057708   .109188
  32.0   .005775   .026732   .062084   .116253
  33.0   .008546   .029948   .066432   .123186
  34.0   .012171   .031092   .070708   .129921
  35.0   .016742   .033198   .074864   .136399
  36.0   .022310   .035244   .078863   .142568
  37.0   .028890   .037210   .082670   .148588
  38.0   .036458   .039080   .086262   .153833

------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 45    A = 5
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  28.0   .000767   .001635   .005842   .016105
  29.0   .001369   .001881   .006654   .018127
  30.0   .002320   .002148   .007520   .020256
  31.0   .003744   .002432   .008435   .022474
  32.0   .005775   .002732   .009387   .024755
  33.0   .008546   .003043   .010366   .027071
  34.0   .012171   .003361   .011358   .029395
  35.0   .016742   .003681   .012350   .031694
  36.0   .022310   .004000   .013328   .033942
  37.0   .028890   .004313   .014282   .036115
  38.0   .036458   .004616   .015201   .038192

------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 45    A = 6
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  28.0   .000767   .000092   .000482   .001871
  29.0   .001369   .000110   .000570   .002192
```

```
  30.0   .002320   .000130   .000669   .002545
  31.0   .003744   .000152   .000776   .002926
  32.0   .005775   .000176   .000891   .003331
  33.0   .008546   .000202   .001013   .003756
  34.0   .012171   .000229   .001140   .004195
  35.0   .016742   .000256   .001270   .004641
  36.0   .022310   .000284   .001401   .005087
  37.0   .028890   .000312   .001532   .005528
  38.0   .036458   .000340   .001660   .005958

------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 45    A = 7
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  28.0   .000767   .000004   .000027   .000148
  29.0   .001369   .000004   .000034   .000180
  30.0   .002320   .000005   .000041   .000216
  31.0   .003744   .000007   .000049   .000257
  32.0   .005775   .000008   .000058   .000302
  33.0   .008546   .000009   .000067   .000350
  34.0   .012171   .000011   .000077   .000401
  35.0   .016742   .000012   .000088   .000454
  36.0   .022310   .000014   .000099   .000508
  37.0   .028890   .000015   .000110   .000562
  38.0   .036458   .000017   .000121   .000616

------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 45    A = 8
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  28.0   .000767   .000000   .000001   .000008
  29.0   .001369   .000000   .000001   .000011
  30.0   .002320   .000000   .000002   .000013
  31.0   .003744   .000000   .000002   .000016
  32.0   .005775   .000000   .000003   .000020
  33.0   .008546   .000000   .000003   .000023
  34.0   .012171   .000000   .000004   .000027
  35.0   .016742   .000000   .000004   .000031
  36.0   .022310   .000000   .000005   .000036
  37.0   .028890   .000001   .000006   .000040
  38.0   .036458   .000001   .000006   .000045

------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 45    A = 9
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  28.0   .000767   .000000   .000000   .000000
  29.0   .001369   .000000   .000000   .000000
  30.0   .002320   .000000   .000000   .000001
  31.0   .003744   .000000   .000000   .000001
  32.0   .005775   .000000   .000000   .000001
  33.0   .008546   .000000   .000000   .000001
  34.0   .012171   .000000   .000000   .000001
  35.0   .016742   .000000   .000000   .000002
  36.0   .022310   .000000   .000000   .000002
  37.0   .028890   .000000   .000000   .000002
  38.0   .036458   .000000   .000000   .000002

------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 45    A = 10
------------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------------
  28.0   .000767   .000000   .000000   .000000
  29.0   .001369   .000000   .000000   .000000
  30.0   .002320   .000000   .000000   .000000
  31.0   .003744   .000000   .000000   .000000
  32.0   .005775   .000000   .000000   .000000
  33.0   .008546   .000000   .000000   .000000
  34.0    .012171    .000000    .000000    .000000
  35.0    .016742    .000000    .000000    .000000
  36.0    .022310    .000000    .000000    .000000
  37.0    .028890    .000000    .000000    .000000
  38.0    .036458    .000000    .000000    .000000
--------------------------------------------------

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 50   A = 5
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  32.0    .000754    .002925    .009649    .025312
  33.0    .001294    .003183    .010765    .027915
  34.0    .002121    .003565    .011940    .030619
  35.0    .003333    .003969     015165    .033402
  36.0    .005036    .004391    .014430    .036238
  37.0    .007335    .004926     015720    .039099
  38.0    .010328    .005269     017023    .041955
  39.0    .014095    .005714    .018322    .044775
  40.0    .018691    .006158    .019604    .047530
  41.0    .024143    .006594    .020855    .050196
  42.0    .030451    .007018    .022064    .052751

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 50   A = 6
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  32.0    .000754    .000186    .000934    .003465
  33.0    .001294    .000217    .001078    .003962
  34.0    .002121    .000250    .001236    .004497
  35.0    .003333    .000287    .001406    .005067
  36.0    .005036    .000326    .001587    .005667
  37.0    .007335    .000368    .001776    .006290
  38.0    .010328    .000411    .001972    .006929
  39.0    .014095    .000456    .002172    .007576
  40.0    .018691    .000501    .002374    .008223
  41.0    .024143    .000545    .002575    .008862
  42.0    .030451    .000591    .002772    .009487

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 50   A = 7
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  32.0    .000754     000008    .000062    .000321
  33.0    .001294    .000010    .000074    .000380
  34.0    .002121    .000012    .000087    .000446
  35.0    .003333    .000014    .000102    .000518
  36.0    .005036    .000017    .000118    .000595
  37.0    .007335    .000019    .000136    .000678
  38.0    .010328    .000022    .000154    .000765
  39.0    .014095    .000025    .000173    .000855
  40.0    .018691    .000028    .000193    .000947
  41.0    .024143    .000031    .000213    .001040
  42.0    .030451    .000034    .000232    .001131

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 50   A = 8
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  32.0    .000754    .000000    .000003    .000021
  33.0    .001294    .000000    .000004    .000026
  34.0    .002121    .000000    .000004    .000032
  35.0    .003333    .000001    .000005    .000038
  36.0    .005036    .000001    .000006    .000045
  37.0    .007335    .000001    .000007    .000052
  38.0    .010328    .000001    .000009    .000060
  39.0    .014095    .000001    .000010    .000068
  40.0    .018691    .000001    .000011    .000077
  41.0    .024143    .000001    .000013    .000086
  42.0    .030451    .000001    .000014    .000095

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 50   A = 9
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  32.0    .000754    .000000     000000    .000001
  33 0    .001294    .000000     000000    .000001
  34.0    .002121    .000000    .000000     000002
  35.0    .003333    .000000    .000000    .000002
  36.0     005036    .000000    .000000    .000003
  37.0    .007335    .000000    .000000    .000003
  38.0    .010328    .000000     000000    .000004
  39.0    .014095    .000000    .000000    .000004
  40.0    .018691    .000000    .000000    .000005
  41.0    .024143    .000000    .000001    .000005
  42.0    .030451    .000000    .000001    .000006

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 50   A = 10
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  32.0    .000754    .000000    .000000    .000000
  33.0    .001294    .000000    .000000    .000000
  34.0    .002121    .000000    .000000    .000000
  35.0    .003333    .000000    .000000    .000000
  36.0    .005036    .000000    .000000    .000000
  37.0    .007335    .000000    .000000    .000000
  38.0    .010328    .000000    .000000    .000000
  39.0    .014095    .000000    .000000    .000000
  40.0    .018691    .000000    .000000    .000000
  41.0    .024143    .000000     000000    .000000
  42.0    .030451    .000000    .000000    .000000
--------------------------------------------------

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 60   A = 5
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  40.0    .000679    .006764    .021187    .050580
  41.0    .001101    .007406    .022966    .054240
  42.0    .001722    .008078    .024804    .057970
  43.0    .002604    .008776    .026690    .061750
  44.0    .003818    .009495    .028613    .065554
  45.0    .005434    .010230    .030556    .069356
  46.0    .007522    .010974    .032505    .073128
  47.0    .010146    .011721    .034443    .076841
  48.0    .013356    .012465    .036355    .080467
  49.0    .017190    .013198    .038225    .083983
  50.0    .021668    .013915    .040039    .087366
  51.0    .026794    .014610    .041787    .090599
  52.0    .032554    .015280    .043458    .093668
  53.0    .038919    .015920    .045046    .096565
  54.0    .045849    .016529    .046547    .099287
  55.0    .053294    .017104    .047959    .101831

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 60   A = 6
--------------------------------------------------
LOAD       OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  40.0    .000679    .000579    .002695    .009147
  41.0    .001101    .000652    .003005    .010103
  42.0    .001722    .000730    .003337    .011109
  43.0    .002604    .000814    .003687    .012158
  44.0    .003818    .000902    .004054    .013244
  45.0    .005434    .000994    .004433    .014357
  46.0    .007522    .001089    .004823    .015490
  47.0    .010146    .001187    .005219    .016630
  48.0    .013356    .001285    .005618    .017767
```

```
   49.0   .017190   .001384   .006015   .018891
   50.0   .021668   .001483   .006406   .019992
   51.0   .026794   .001579   .006790   .021062
   52.0   .032554   .001674   .007161   .022094
   53.0   .038919   .001765   .007520   .023082
   54.0   .045849   .001853   .007863   .024023
   55.0   .053294   .001937   .008189   .024913

---------------------------------------------------
BLOCKING PROBABILITIES FOR C = 60   A = 7
---------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
---------------------------------------------------
   40.0   .000679   .000034   .000232   .001115
   41.0   .001101   .000039   .000266   .001267
   42.0   .001722   .000045   .000303   .001432
   43.0   .002604   .000052   .000343   .001609
   44.0   .003818   .000058   .000386   .001797
   45.0   .005434   .000066   .000432   .001994
   46.0   .007522   .000073   .000480   .002199
   47.0   .010146   .000081   .000529   .002409
   48.0   .013356   .000090   .000580   .002623
   49.0   .017190   .000098   .000631   .002838
   50.0   .021668   .000106   .000682   .003052
   51.0   .026794   .000115   .000733   .003264
   52.0   .032554   .000123   .000783   .003470
   53.0   .038919   .000131   .000832   .003671
   54.0   .045849   .000139   .000879   .003864
   55.0   .053294   .000147   .000924   .004049

---------------------------------------------------
BLOCKING PROBABILITIES FOR C = 60   A = 8
---------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
---------------------------------------------------
   40.0   .000679   .000001   .000014   .000097
   41.0   .001101   .000002   .000017   .000113
   42.0   .001722   .000002   .000020   .000131
   43.0   .002604   .000002   .000023   .000151
   44.0   .003818   .000003   .000026   .000173
   45.0   .005434   .000003   .000030   .000196
   46.0   .007522   .000004   .000034   .000220
   47.0   .010146   .000004   .000038   .000246
   48.0   .013356   .000005   .000043   .000272
   49.0   .017190   .000005   .000047   .000299
   50.0   .021668   .000005   .000052   .000327
   51.0   .026794   .000006   .000056   .000354
   52.0   .032554   .000006   .000061   .000381
   53.0   .038919   .000007   .000065   .000407
   54.0   .045849   .000007   .000069   .000433
   55.0   .053294   .000008   .000073   .000458

---------------------------------------------------
BLOCKING PROBABILITIES FOR C = 60   A = 9
---------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
---------------------------------------------------
   40.0   .000679   .000000   .000001   .000006
   41.0   .001101   .000000   .000001   .000008
   42.0   .001722   .000000   .000001   .000009
   43.0   .002604   .000000   .000001   .000011
   44.0   .003818   .000000   .000001   .000012
   45.0   .005434   .000000   .000002   .000014
   46.0   .007522   .000000   .000002   .000016
   47.0   .010146   .000000   .000002   .000019
   48.0   .013356   .000000   .000002   .000021
   49.0   .017190   .000000   .000003   .000023
   50.0   .021668   .000000   .000003   .000026
   51.0   .026794   .000000   .000003   .000028
   52.0   .032554   .000000   .000003   .000031
   53.0   .038919   .000000   .000004   .000033
   54.0   .045849   .000000   .000004   .000036
   55.0   .053294   .000000   .000004   .000038

---------------------------------------------------
BLOCKING PROBABILITIES FOR C = 60   A = 10
---------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
---------------------------------------------------
   40.0   .000679   .000000   .000000    000000
   41.0   .001101   .000000   .000000   .000000
   42.0   .001722    000000   .000000   .000000
   43.0   .002604   .000000   .000000   .000001
   44.0   .003818   .000000   .000000   .000001
   45.0   .005434   .000000   .000000   .000001
   46.0   .007522   .000000   .000000   .000001
   47.0   .010146   .000000   .000000   .000001
   48.0   .013356   .000000   .000000   .000001
   49.0    017190   .000000   .000000   .000001
   50.0   .021668   .000000   .000000   .000002
   51.0   .026794   .000000   .000000   .000002
   52.0   .032554   .000000   .000000   .000002
   53.0   .038919   .000000   .000000   .000002
   54.0   .045849   .000000   .000000   .000002
   55.0   .053294   .000000   .000000   .000002

---------------------------------------------------
BLOCKING PROBABILITIES FOR C = 60   A = 11
---------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
---------------------------------------------------
   40.0   .000679   .000000   .000000   .000000
   41.0   .001101   .000000   .000000   .000000
   42.0   .001722   .000000   .000000   .000000
   43.0   .002604   .000000   .000000   .000000
   44.0   .003818   .000000   .000000   .000000
   45.0   .005434   .000000   .000000   .000000
   46.0   .007522   .000000   .000000   .000000
   47.0   .010146   .000000   .000000   .000000
   48.0   .013356   .000000   .000000   .000000
   49.0   .017190   .000000   .000000   .000000
   50.0   .021668   .000000   .000000   .000000
   51.0   .026794   .000000   .000000   .000000
   52.0   .032554   .000000   .000000   .000000
   53.0   .038919   .000000   .000000   .000000
   54.0   .045849   .000000   .000000   .000000
   55.0   .053294   .000000   .000000   .000000
---------------------------------------------------

---------------------------------------------------
BLOCKING PROBABILITIES FOR C = 70   A = 5
---------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
---------------------------------------------------
   50.0   .001368   .015217   .043016   .092368
   51.0   .002016   .016257   .045527   .096856
   52.0   .002895   .017318   .048067   .101339
   53.0   .004054   .018397   .050619   .105795
   54.0   .005545   .019485   .053168   .110199
   55.0   .007417   .020575   .055699   .114528
   56.0   .009714   .021660   .058194   .118757
   57.0   .012474   .022733   .060639   .122865
   58.0   .015723   .023785   .063019   .126831
   59.0   .019478   .024811   .065321   .130639
   60.0   .023744   .025805   .067536   .134274
   61.0   .028517   .026761   .069654   .137728
   62.0   .033779   .027677   .071669   .140994
   63.0   .039506   .028550   .073578   .144068
   64.0   .045668   .029378   .075379   .146953
   65.0   .052227   .030160   .077072   .149651

---------------------------------------------------
BLOCKING PROBABILITIES FOR C = 70   A = 6
---------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
```

```
--------------------------------------------------
  50.0    .001368   .001703   .007205   .022015
  51.0    .002016   .001860   .007805   .023619
  52.0    .002895   .002024   .008425   .025262
  53.0    .004054   .002194   .009062   .026931
  54.0    .005545   .002370   .009712   .028617
  55.0    .007417   .002549   .010370   .030309
  56.0    .009714   .002730   .011031   .031992
  57.0    .012474   .002911   .011690   .033657
  58.0    .015723   .003093   .012342   .035291
  59.0    .019478   .003272   .012982   .036584
  60.0    .023744   .003447   .013607   .038428
  61.0    .028517   .003619   .014213   .039913
  62.0    .033779   .003784   .014796   .041336
  63.0    .039506   .003944   .015355   .042691
  64.0    .045668   .004097   .015988   .043977
  65.0    .052227   .004243   .016394   .045192

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 70    A = 7
--------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  50.0    .001368   .000129   .000811   .003543
  51.0    .002016   .000144   .000898   .003890
  52.0    .002895   .000160   .000990   .004253
  53.0    .004054   .000177   .001087   .004631
  54.0    .005545   .000194   .001187   .005020
  55.0    .007417   .000213   .001290   .005418
  56.0    .009714   .000231   .001396   .005821
  57.0    .012474   .000250   .001503   .006227
  58.0    .015723   .000270   .001610   .006631
  59.0    .019478   .000289   .001717   .007031
  60.0    .023744   .000308   .001822   .007424
  61.0    .028517   .000327   .001926   .007807
  62.0    .033779   .000345   .002026   .008179
  63.0    .039506   .000363   .002124   .008536
  64.0    .045668   .000380   .002218   .008879
  65.0    .052227   .000397   .002307   .009206

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 70    A = 8
--------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  50.0    .001368   .000007   .000065   .000404
  51.0    .002016   .000008   .000074   .000453
  52.0    .002895   .000009   .000083   .000505
  53.0    .004054   .000010   .000093   .000561
  54.0    .005545   .000011   .000103   .000620
  55.0    .007417   .000013   .000114   .000681
  56.0    .009714   .000014   .000125   .000743
  57.0    .012474   .000015   .000137   .000807
  58.0    .015723   .000017   .000148   .000872
  59.0    .019478   .000018   .000160   .000937
  60.0    .023744   .000020   .000172   .001001
  61.0    .028517   .000021   .000183   .001065
  62.0    .033779   .000022   .000195   .001127
  63.0    .039506   .000024   .000206   .001188
  64.0    .045668   .000025   .000217   .001246
  65.0    .052227   .000026   .000227   .001302

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 70    A = 9
--------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  50.0    .001368   .000000   .000004   .000034
  51.0    .002016   .000000   .000005   .000039
  52.0    .002895   .000000   .000005   .000045
  53.0    .004054   .000000   .000006   .000050
  54.0    .005545   .000001   .000007   .000057
  55.0    .007417   .000001   .000007   .000063
  56.0    .009714   .000001   .000008   .000070
  57.0    .012474   .000001   .000009   .000077
  58.0    .015723   .000001   .000010   .000084
  59.0    .019478   .000001   .000011   .000092
  60.0    .023744   .000001   .000012   .000099
  61.0    .028517   .000001   .000013   .000106
  62.0    .033779   .000001   .000014   .000113
  63.0    .039506   .000001   .000015   .000121
  64.0    .045668   .000001   .000016   .000127
  65.0    .052227   .000001   .000017   .000134

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 70    A = 10
--------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  50.0    .001368   .000000   .000000   .000002
  51.0    .002016   .000000   .000000   .000003
  52.0    .002895   .000000   .000000   .000003
  53.0    .004054   .000000   .000000   .000003
  54.0    .005545   .000000   .000000   .000004
  55.0    .007417   .000000   .000000   .000004
  56.0    .009714   .000000   .000000   .000005
  57.0    .012474   .000000   .000000   .000006
  58.0    .015723   .000000   .000001   .000006
  59.0    .019478   .000000   .000001   .000007
  60.0    .023744   .000000   .000001   .000007
  61.0    .028517   .000000   .000001   .000008
  62.0    .033779   .000000   .000001   .000009
  63.0    .039506   .000000   .000001   .000009
  64.0    .045668   .000000   .000001   .000010
  65.0    .052227   .000000   .000001   .000010

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 70    A = 11
--------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  50.0    .001368   .000000   .000000   .000000
  51.0    .002016   .000000   .000000   .000000
  52.0    .002895   .000000   .000000   .000000
  53.0    .004054   .000000   .000000   .000000
  54.0    .005545   .000000   .000000   .000000
  55.0    .007417   .000000   .000000   .000000
  56.0    .009714   .000000   .000000   .000000
  57.0    .012474   .000000   .000000   .000000
  58.0    .015723   .000000   .000000   .000000
  59.0    .019478   .000000   .000000   .000000
  60.0    .023744   .000000   .000000   .000000
  61.0    .028517   .000000   .000000   .000000
  62.0    .033779   .000000   .000000   .000001
  63.0    .039506   .000000   .000000   .000001
  64.0    .045668   .000000   .000000   .000001
  65.0    .052227   .000000   .000000   .000001
--------------------------------------------------
--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 80    A = 6
--------------------------------------------------
LOAD        OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  60.0    .002199   .003922   .015139   .041935
  61.0    .003043   .004199   .016081   .044050
  62.0    .004126   .004483   .017036   .046275
  63.0    .005485   .004773   .018001   .048501
  64.0    .007158   .005066   .018970   .050713
  65.0    .009176   .005360   .019935   .052901
  66.0    .011570   .005654   .020892   .055051
  67.0    .014358   .005945   .021835   .057153
  68.0    .017557   .006232   .022758   .059196
  69.0    .021172   .006514   .023657   .061171
  70.0    .025203   .006788   .024528   .063072
```

```
   71.0    .029640    .007053    .025367    .064893
   72.0    .034468    .007310    .026171    .066629
   73.0    .039668    .007555    .026940    .068278
   74.0    .045214    .007790    .027672    .069840
   75.0    .051078    .008015    .028366    .071315

--------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 80    A = 7
--------------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------------
   60.0    .002199    .000369    .002135    .008483
   61.0    .003043    .000402    .002310    .009103
   62.0    .004126    .000437    .002491    .009738
   63.0    .005485    .000472    .002677    .010385
   64.0    .007158    .000509    .002866    .011039
   65.0    .009176    .000546    .003057    .011697
   66.0    .011570    .000584    .003250    .012352
   67.0    .014358    .000622    .003442    .013003
   68.0    .017557    .000659    .003633    .013643
   69.0    .021172    .000696    .003820    .014271
   70.0    .025203    .000733    .004004    .014882
   71.0    .029640    .000769    .004183    .015473
   72.0    .034468    .000804    .004356    .016043
   73.0    .039668    .000837    .004523    .016590
   74.0    .045214    .000870    .004684    .017112
   75.0    .051078    .000901    .004837    .017610

--------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 80    A = 8
--------------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------------
   60.0    .002199    .000025    .000213    .001212
   61.0    .003043    .000028    .000235    .001324
   62.0    .004126    .000031    .000257    .001441
   63.0    .005485    .000033    .000281    .001563
   64.0    .007158    .000037    .000305    .001687
   65.0    .009176    .000040    .000330    .001814
   66.0    .011570    .000043    .000356    .001943
   67.0    .014358    .000046    .000381    .002072
   68.0    .017557    .000050    .000407    .002201
   69.0    .021172    .000053    .000432    .002328
   70.0    .025203    .000056    .000458    .002453
   71.0    .029640    .000059    .000482    .002576
   72.0    .034468    .000063    .000506    .002695
   73.0    .039668    .000066    .000530    .002811
   74.0    .045214    .000069    .000553    .002922
   75.0    .051078    .000072    .000575    .003028

--------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 80    A = 9
--------------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------------
   60.0    .002199    .000001    .000016    .000128
   61.0    .003043    .000001    .000018    .000142
   62.0    .004126    .000002    .000020    .000157
   63.0    .005485    .000002    .000022    .000173
   64.0    .007158    .000002    .000024    .000189
   65.0    .009176    .000002    .000026    .000206
   66.0    .011570    .000002    .000029    .000224
   67.0    .014358    .000003    .000031    .000241
   68.0    .017557    .000003    .000034    .000259
   69.0    .021172    .000003    .000036    .000277
   70.0    .025203    .000003    .000039    .000294
   71.0    .029640    .000003    .000041    .000312
   72.0    .034468    .000004    .000043    .000329
   73.0    .039668    .000004    .000046    .000346
   74.0    .045214    .000004    .000048    .000362
   75.0    .051078    .000004    .000050    .000377

--------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 80    A = 10
--------------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------------
   60.0    .002199    .000000    .000001    .000010
   61.0    .003043    .000000    .000001    .000012
   62.0    .004126    .000000    .000001    .000013
   63.0    .005485    .000000    .000001    .000015
   64.0    .007158    .000000    .000001    .000016
   65.0    .009176    .000000    .000002    .000018
   66.0    .011570    .000000    .000002    .000020
   67.0    .014358    .000000    .000002    .000021
   68.0    .017557    .000000    .000002    .000023
   69.0    .021172    .000000    .000002    .000025
   70.0    .025203    .000000    .000002    .000027
   71.0    .029640    .000000    .000003    .000029
   72.0    .034468    .000000    .000003    .000030
   73.0    .039668    .000000    .000003    .000032
   74.0    .045214    .000000    .000003    .000034
   75.0    .051078    .000000    .000003    .000036

--------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 80    A = 11
--------------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------------
   60.0    .002199    .000000    .000000    .000001
   61.0    .003043    .000000    .000000    .000001
   62.0    .004126    .000000    .000000    .000001
   63.0    .005485    .000000    .000000    .000001
   64.0    .007158    .000000    .000000    .000001
   65.0    .009176    .000000    .000000    .000001
   66.0    .011570    .000000    .000000    .000001
   67.0    .014358    .000000    .000000    .000001
   68.0    .017557    .000000    .000000    .000002
   69.0    .021172    .000000    .000000    .000002
   70.0    .025203    .000000    .000000    .000002
   71.0    .029640    .000000    .000000    .000002
   72.0    .034468    .000000    .000000    .000002
   73.0    .039668    .000000    .000000    .000002
   74.0    .045214    .000000    .000000    .000002
   75.0    .051078    .000000    .000000    .000003

--------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 80    A = 12
--------------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------------
   60.0    .002199    .000000    .000000    .000000
   61.0    .003043    .000000    .000000    .000000
   62.0    .004126    .000000    .000000    .000000
   63.0    .005485    .000000    .000000    .000000
   64.0    .007158    .000000    .000000    .000000
   65.0    .009176    .000000    .000000    .000000
   66.0    .011570    .000000    .000000    .000000
   67.0    .014358    .000000    .000000    .000000
   68.0    .017557    .000000    .000000    .000000
   69.0    .021172    .000000    .000000    .000000
   70.0    .025203    .000000    .000000    .000000
   71.0    .029640    .000000    .000000    .000000
   72.0    .034468    .000000    .000000    .000000
   73.0    .039668    .000000    .000000    .000000
   74.0    .045214    .000000    .000000    .000000
   75.0    .051078    .000000    .000000    .000000
--------------------------------------------------------
--------------------------------------------------------
BLOCKING PROBABILITIES FOR C = 90    A = 6
--------------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------------
   70.0    .003092    .007640    .026982    .067914
```

```
  71.0    .004092   .008066   .028273   .070600
  72.0    .005325   .008497   .029566   .073268
  73.0    .006820   .008930   .030855   .075906
  74.0    .008603   .009363   .032135   .078502
  75.0    .010695   .009794   .033398   .081044
  76.0    .013115   .010220   .034639   .083522
  77.0    .015874   .010639   .035851   .085927
  78.0    .018980   .011049   .037030   .088251
  79.0    .022434   .011448   .038172   .090487
  80.0    .026232   .011835   .039273   .092629
  81.0    .030365   .012209   .040330   .094675
  82.0    .034819   .012569   .041342   .096623
  83.0    .039577   .012913   .042307   .098470
  84.0    .044620   .013242   .043224   .100219
  85.0    .049926   .013556   .044094   .101871

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 90    A = 7
-----------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
  70.0    .003092   .000867   .004622   .016741
  71.0    .004092   .000929   .004920   .017681
  72.0    .005325   .000993   .005223   .018629
  73.0    .006820   .001058   .005529   .019581
  74.0    .008603   .001124   .005837   .020532
  75.0    .010695   .001190   .006146   .021475
  76.0    .013115   .001256   .006452   .022407
  77.0    .015874   .001321   .006755   .023322
  78.0    .018980   .001386   .007053   .024216
  79.0    .022434   .001450   .007344   .025085
  80.0    .026232   .001512   .007628   .025927
  81.0    .030365   .001573   .007903   .026739
  82.0    .034819   .001632   .008168   .027518
  83.0    .039577   .001689   .008423   .028264
  84.0    .044620   .001743   .008668   .028976
  85.0    .049926   .001796   .008902   .029653

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 90    A = 8
-----------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
  70.0    .003092   .000070   .000558   .002909
  71.0    .004092   .000076   .000603   .003121
  72.0    .005325   .000083   .000649   .003338
  73.0    .006820   .000089   .000696   .003558
  74.0    .008603   .000096   .000744   .003782
  75.0    .010695   .000102   .000793   .004006
  76.0    .013115   .000109   .000842   .004230
  77.0    .015874   .000116   .000890   .004453
  78.0    .018980   .000123   .000939   .004673
  79.0    .022434   .000130   .000987   .004890
  80.0    .026232   .000136   .001034   .005101
  81.0    .030365   .000143   .001080   .005306
  82.0    .034819   .000149   .001124   .005505
  83.0    .039577   .000155   .001168   .005697
  84.0    .044620   .000161   .001209   .005882
  85.0    .049926   .000167   .001250   .006059

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 90    A = 9
-----------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
  70.0    .003092   .000004   .000050   .000370
  71.0    .004092   .000005   .000055   .000403
  72.0    .005325   .000005   .000060   .000437
  73.0    .006820   .000006   .000065   .000472
  74.0    .008603   .000006   .000070   .000508
  75.0    .010695   .000007   .000075   .000545
  76.0    .013115   .000007   .000081   .000582
  77.0    .015874   .000008   .000086   .000618
  78.0    .018980   .000008   .000092   .000655
  79.0    .022434   .000009   .000097   .000692
  80.0    .026232   .000009   .000103   .000728
  81.0    .030365   .000010   .000108   .000763
  82.0    .034819   .000010   .000113   .000798
  83.0    .039577   .000011   .000118   .000831
  84.0    .044620   .000011   .000123   .000863
  85.0    .049926   .000012   .000128   .000895

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 90    A = 10
-----------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
  70.0    .003092   .000000   .000003   .000036
  71.0    .004092   .000000   .000004   .000040
  72.0    .005325   .000000   .000004   .000044
  73.0    .006820   .000000   .000005   .000048
  74.0    .008603   .000000   .000005   .000052
  75.0    .010695   .000000   .000005   .000056
  76.0    .013115   .000000   .000006   .000061
  77.0    .015874   .000000   .000006   .000065
  78.0    .018980   .000000   .000007   .000070
  79.0    .022434   .000000   .000007   .000074
  80.0    .026232   .000000   .000008   .000078
  81.0    .030365   .000001   .000008   .000083
  82.0    .034819   .000001   .000009   .000087
  83.0    .039577   .000001   .000009   .000091
  84.0    .044620   .000001   .000010   .000095
  85.0    .049926   .000001   .000010   .000099

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 90    A = 11
-----------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
  70.0    .003092   .000000   .000000   .000003
  71.0    .004092   .000000   .000000   .000003
  72.0    .005325   .000000   .000000   .000003
  73.0    .006820   .000000   .000000   .000004
  74.0    .008603   .000000   .000000   .000004
  75.0    .010695   .000000   .000000   .000005
  76.0    .013115   .000000   .000000   .000005
  77.0    .015874   .000000   .000000   .000005
  78.0    .018980   .000000   .000000   .000006
  79.0    .022434   .000000   .000000   .000006
  80.0    .026232   .000000   .000000   .000007
  81.0    .030365   .000000   .000000   .000007
  82.0    .034819   .000000   .000001   .000007
  83.0    .039577   .000000   .000001   .000008
  84.0    .044620   .000000   .000001   .000008
  85.0    .049926   .000000   .000001   .000009

-----------------------------------------------------
BLOCKING PROBABILITIES FOR C = 90    A = 12
-----------------------------------------------------
LOAD        OUTER   INNER(28)  INNER(35)  INNER(42)
-----------------------------------------------------
  70.0    .003092   .000000   .000000   .000000
  71.0    .004092   .000000   .000000   .000000
  72.0    .005325   .000000   .000000   .000000
  73.0    .006820   .000000   .000000   .000000
  74.0    .008603   .000000   .000000   .000000
  75.0    .010695   .000000   .000000   .000000
  76.0    .013115   .000000   .000000   .000000
  77.0    .015874   .000000   .000000   .000000
  78.0    .018980   .000000   .000000   .000000
  79.0    .022434   .000000   .000000   .000000
  80.0    .026232   .000000   .000000   000000
  81.0    .030365   .000000   .000000   .000000
  82.0    .034819   .000000   .000000   .000001
  83.0   .039577   .000000   .000000   .000001
  84.0   .044620   .000000   .000000   .000001
  85.0   .049926   .000000   .000000   .000001
--------------------------------------------------

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 100    A = 7
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  80.0   .003992   .001765   .008680   .028751
  81.0   .005109   .001869   .009129   .030016
  82.0   .006449   .001973   .009580   .031279
  83.0   .008030   .002079   .010032   .032536
  84.0   .009873   .002185   .010482   .033775
  85.0   .011990   .002290   .010929   .034997
  86.0   .014395   .002395   .011369   .036195
  87.0   .017093   .002498   .011802   .037364
  88.0   .020088   .002600   .012224   .038500
  89.0   .023378   .002699   .012636   .039600
  90.0   .026957   .002796   .013035   .040661
  91.0   .030818   .002890   .013421   .041681
  92.0   .034948   .002981   .013792   .042658
  93.0   .039334   .003068   .014149   .043592
  94.0   .043958   .003152   .014490   .044482
  95.0   .048804   .003233   .014816   .045328

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 100    A = 8
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  80.0   .003992   .000167   .001238   .005937
  81.0   .005109   .000179   .001319   .006283
  82.0   .006449   .000192   .001401   .006632
  83.0   .008030   .000204   .001484   .006984
  84.0   .009873   .000217   .001568   .007336
  85.0   .011990   .000229   .001653   .007687
  86.0   .014395   .000242   .001736   .008034
  87.0   .017093   .000255   .001819   .008376
  88.0   .020088   .000267   .001901   .008712
  89.0   .023378   .000280   .001982   .009040
  90.0   .026957   .000292   .002060   .009359
  91.0   .030818   .000303   .002137   .009668
  92.0   .034948   .000315   .002211   .009967
  93.0   .039334   .000326   .002282   .010254
  94.0   .043958   .000337   .002351   .010530
  95.0   .048904   .000347   .002418   .010794

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 100    A = 9
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  80.0   .003992   .000012   .000130   .000895
  81.0   .005109   .000013   .000140   .000959
  82.0   .006449   .000014   .000151   .001025
  83.0   .008030   .000015   .000161   .001092
  84.0   .009873   .000016   .000172   .001159
  85.0   .011990   .000017   .000183   .001228
  86.0   .014395   .000018   .000194   .001296
  87.0   .017093   .000019   .000205   .001363
  88.0   .020088   .000020   .000216   .001430
  89.0   .023378   .000021   .000227   .001496
  90.0   .026957   .000023   .000237   .001561
  91.0   .030818   .000024   .000248   .001624
  92.0   .034948   .000025   .000258   .001685
  93.0   .039334   .000026   .000268   .001745
  94.0   .043958   .000027   .000277   .001802
  95.0   .048804   .000028   .000287   .001857

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 100    A = 10

--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  80.0   .003992   .000001   .000010   .000102
  81.0   .005109   .000001   .000011   .000111
  82.0   .006449   .000001   .000012   .000120
  83.0   .008030   .000001   .000013   .000129
  84.0   .009873   .000001   .000014   .000138
  85.0   .011990   .000001   .000015   .000148
  86.0   .014395   .000001   .000017   .000158
  87.0    017093   .000001   .000018   .000167
  88.0   .020088   .000001   .000019   .000177
  89.0   .023378   .000001   .000020   .000186
  90.0   .026957   .000001   .000021   .000196
  91.0   .030818   .000001   .000022   .000205
  92.0   .034948   .000001   .000023   .000214
  93.0   .039334   .000002   .000024   .000223
  94.0   .043958   .000002   .000025   .000231
  95.0   .048804   .000002   .000026   .000239

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 100    A = 11
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  80.0   .003992   .000000   .000001   .000009
  81.0   .005109   .000000   .000001   .000010
  82.0   .006449   .000000   .000001   .000011
  83.0   .008030   .000000   .000001   .000012
  84.0   .009873   .000000   .000001   .000013
  85.0   .011990   .000000   .000001   .000014
  86.0   .014395   .000000   .000001   .000015
  87.0   .017093   .000000   .000001   .000016
  88.0   .020088   .000000   .000001   .000017
  89.0   .023378   .000000   .000001   .000018
  90.0   .026957   .000000   .000001   .000019
  91.0   .030818   .000000   .000002   .000020
  92.0   .034948   .000000   .000002   .000021
  93.0   .039334   .000000   .000002   .000022
  94.0   .043958   .000000   .000002   .000023
  95.0   .048804   .000000   .000002   .000024

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 100    A = 12
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  80.0   .003992   .000000   .000000   .000001
  81.0   .005109   .000000   .000000   .000001
  82.0   .006449   .000000   .000000   .000001
  83.0   .008030   .000000   .000000   .000001
  84.0   .009873   .000000   .000000   .000001
  85.0   .011990   .000000   .000000   .000001
  86.0   .014395   .000000   .000000   .000001
  87.0   .017093   .000000   .000000   .000001
  88.0   .020088   .000000   .000000   .000001
  89.0   .023378   .000000   .000000   .000001
  90.0   .026957   .000000   .000000   .000001
  91.0   .030818   .000000   .000000   .000002
  92.0   .034948   .000000   .000000   .000002
  93.0   .039334   .000000   .000000   .000002
  94.0   .043958   .000000    000000   .000002
  95.0   .048804   .000000   .000000   .000002

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 100    A = 13
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
  80.0   .003992   .000000   .000000   .000000
  81.0   .005109   .000000   .000000   .000000
  82.0   .006449   .000000   .000000   .000000
```

```
 83.0   .008030   000000   .000000   .000000
 84.0   .009873   .000000  .000000   .000000
 85.0   .011990   .000000  .000000   .000000
 86.0   .014395   .000000  .000000   .000000
 87.0   .017093   .000000  .000000   .000000
 88.0   .020088   .000000  .000000   .000000
 89.0   .023378   .000000  .000000   .000000
 90.0   .026957   .000000  .000000   .000000
 91.0   .030818   .000000  .000000   .000000
 92.0    034948   .000000  .000000   .000000
 93.0   .039334   .000000  .000000   .000000
 94.0   .043958   .000000  .000000   .000000
 95.0    048804   .000000  .000000   .000000
------------------------------------------------
------------------------------------------------
BLOCKING PROBABILITIES FOR C = 125    A = 7
------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------
 100.0   .001989   .005526   .023114   .064527
 101.0   .002544   .005776   .023994   .066509
 102.0   .003217   .006029   .024879   .068486
 103.0   .004021   .006285   .025766   .070452
 104.0   .004971   .006542   .026652   .072401
 105.0   .006082   .006800   .027534   .074328
 106.0    007366   .007057   .028410   .076226
 107.0   .008835   .007314   .029276   .078091
 108.0   .010498   .007568   .030129   .079917
 109.0   .012363   .007819   .030966   .081699
 110.0   .014434   .008066   .031786   .083433
 111.0   .016717   .008308   .032586   .085115
 112.0   .019210   .008545   .033365   .086742
 113.0   .021913   .008775   .034119   .088312
 114.0   .024823   .008999   .034849   .089823
 115.0   .027934   .009217   .035553   .091274

------------------------------------------------
BLOCKING PROBABILITIES FOR C = 125    A = 8
------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------
 100.0   .001989   .000696   .004431   .017927
 101.0   .002544   .000736   .004653   .018693
 102.0   .003217   .000776   .004879   .019465
 103.0   .004021   .000817   .005108   .020243
 104.0   .004971   .000859   .005339   .021022
 105.0   .006082   .000902   .005572   .021801
 106.0   .007366   .000944   .005805   .022575
 107.0   .008835   .000987   .006038   .023344
 108.0   .010498   .001030   .006269   .024103
 109.0   .012363   .001072   .006498   .024850
 110.0   .014434   .001115    006724   .025584
 111.0   .016717   .001156   .006946   .026301
 112.0   .019210   .001197   .007164   .027000
 113.0   .021913   .001238   .007377   .027679
 114.0   .024823   .001277   .007584   .028337
 115.0   .027934   .001315   .007784   .028973

------------------------------------------------
BLOCKING PROBABILITIES FOR C = 125    A = 9
------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------
 100.0   .001989   .000065   .000620   .003647
 101.0   .002544   .000069   .000659   .003848
 102.0   .003217   .000074   .000698   .004052
 103.0   .004021   .000078   .000739   .004260
 104.0   .004971   .000083   .000780   .004471
 105.0   .006082   .000088   .000821   .004683
 106.0   .007366   .000093   .000864   .004897
 107.0   .008835   .000098   .000906   .005110
 108.0   .010498   .000103   .000949   .005324
 109.0   .012363   .000108   .000991   .005535
 110.0   .014434   .000113   .001033   .005744
 111.0   .016717   .000118   .001075   .005950
 112.0   .019210   .000123   .001116   .006153
 113.0   .021913   .000128   .001157   .006351
 114.0   .024823   .000133   .001196   .006544
 115.0   .027934   .000137   .001235   .006732

------------------------------------------------
BLOCKING PROBABILITIES FOR C = 125    A = 10
------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------
 100.0   .001989   .000005   .000066   .000559
 101.0   .002544   .000005   .000071   .000596
 102.0   .003217   .000005   .000076   .000635
 103.0   .004021   .000006   .000081   .000674
 104.0   .004971   .000006   .000086   .000714
 105.0   .006082   .000007   .000092   .000756
 106.0   .007366   .000007   .000097   .000797
 107.0   .008835   .000007   .000103   .000839
 108.0   .010498   .000008   .000109   .000881
 109.0   .012363   .000008   .000114   .000924
 110.0   .014434   .000009   .000120   .000966
 111.0   .016717   .000009   .000126   .001008
 112.0   .019210   .000010   .000131   .001049
 113.0   .021913   .000010   .000137   .001089
 114.0   .024823   .000011   .000142   .001129
 115.0   .027934   .000011   .000148   .001168

------------------------------------------------
BLOCKING PROBABILITIES FOR C = 125    A = 11
------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------
 100.0   .001989   .000000   .000006   .000067
 101.0   .002544   .000000   .000006   .000072
 102.0   .003217   .000000   .000006   .000077
 103.0   .004021   .000000   .000007   .000083
 104.0   .004971   .000000   .000008   .000089
 105.0   .006082   .000000   .000008   .000095
 106.0   .007366   .000000   .000009   .000101
 107.0   .008835   .000000   .000009   .000107
 108.0   .010498   .000000   .000010   .000113
 109.0   .012363   .000001   .000010   .000119
 110.0   .014434   .000001   .000011   .000126
 111.0   .016717   .000001   .000011   .000132
 112.0   .019210   .000001   .000012   .000138
 113.0   .021913   .000001   .000013   .000144
 114.0   .024823   .000001   .000013   .000150
 115.0   .027934   .000001   .000014   .000156

------------------------------------------------
BLOCKING PROBABILITIES FOR C = 125    A = 12
------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
------------------------------------------------
 100.0   .001989   .000000   .000000   .000006
 101.0   .002544   .000000   .000000   .000007
 102.0   .003217   .000000   .000000   .000008
 103.0   .004021   .000000   .000000   .000008
 104.0   .004971   .000000   .000001   .000009
 105.0   .006082   .000000   .000001   .000009
 106.0   .007366   .000000   .000001   .000010
 107.0   .008835   .000000   .000001   .000011
 108.0   .010498   .000000   .000001   .000012
 109.0   .012363   .000000   .000001   .000012
 110.0   .014434   .000000   .000001   .000013
 111.0   .016717   .000000   .000001   .000014
 112.0   .019210   .000000   .000001   .000014
 113.0   .021913   .000000   .000001   .000015
 114.0   .024823   .000000   .000001   .000016
 115.0   .027934   .000000   .000001   .000017

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 125    A = 13
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 100.0   .001989   .000000   .000000   .000001
 101.0   .002544   .000000   .000000   .000001
 102.0   .003217   .000000   .000000   .000001
 103.0   .004021   .000000   .000000   .000001
 104.0   .004971   .000000   .000000   .000001
 105.0   .006092   .000000   .000000   .000001
 106.0   .007366   .000000   .000000   .000001
 107.0   .008835   .000000   .000000   .000001
 108.0   .010498   .000000   .000000   .000001
 109.0   .012363   .000000   .000000   .000001
 110.0   .014434   .000000   .000000   .000001
 111.0   .016717   .000000   .000000   .000001
 112.0   .019210   .000000   .000000   .000001
 113.0   .021913   .000000   .000000   .000001
 114.0   .024823   .000000   .000000   .000001
 115.0   .027934   .000000   .000000   .000001
-------------------------------------------------

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 150    A = 8
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 125.0   .003152   .002545   .013575   .045304
 126.0   .003844   .002647   .014038   .046555
 127.0   .004649   .002750   .014501   .047798
 128.0   .005574   .002854   .014964   .049032
 129.0   .006630   .002957   .015425   .050252
 130.0   .007823   .003061   .015882   .051457
 131.0   .009162   .003164   .016335   .052642
 132.0   .010651   .003266   .016782   .053805
 133.0   .012295   .003367   .017222   .054944
 134.0   .014097   .003466   .017654   .056056
 135.0   .016059   .003564   .018077   .057138

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 150    A = 9
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 125.0   .003152   .000312   .002537   .012372
 126.0   .003844   .000327   .002646   .012829
 127.0   .004649   .000342   .002757   .013287
 128.0   .005574   .000358   .002868   .013745
 129.0   .006630   .000374   .002980   .014203
 130.0   .007823   .000389   .003092   .014657
 131.0   .009162   .000405   .003204   .015109
 132.0   .010651   .000421   .003315   .015555
 133.0   .012295   .000437   .003425   .015994
 134.0   .014097   .000452   .003533   .016426
 135.0   .016059   .000468   .003640   .016850

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 150    A = 10
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 125.0   .003152   .000029   .000357   .002541
 126.0   .003844   .000031   .000376   .002658
 127.0   .004649   .000032   .000395   .002776
 128.0   .005574   .000034   .000414   .002896
 129.0   .006630   .000036   .000433   .003016
 130.0   .007823   .000038   .000452   .003137
 131.0   .009162   .000039   .000472   .003257
 132.0   .010651   .000041   .000492   .003377
 133.0   .012295   .000043   .000511   .003496
 134.0   .014097   .000045   .000530   .003614
 135.0   .016059   .000047   .000550   .003730

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 150    A = 11
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 125.0   .003152   .000002   .000039   .000403
 126.0   .003844   .000002   .000042   .000425
 127.0   .004649   .000002   .000044   .000447
 128.0   .005574   .000003   .000046   .000470
 129.0   .006630   .000003   .000049   .000493
 130.0   .007823   .000003   .000052   .000517
 131.0   .009162   .000003   .000054   .000540
 132.0   .010651   .000003   .000057   .000564
 133.0   .012295   .000003   .000059   .000587
 134.0   .014097   .000003   .000062   .000611
 135.0   .016059   .000004   .000064   .000634

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 150    A = 12
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 125.0   .003152   .000000   .000003   .000051
 126.0   .003844   .000000   .000004   .000054
 127.0   .004649   .000000   .000004   .000057
 128.0   .005574   .000000   .000004   .000061
 129.0   .006630   .000000   .000004   .000064
 130.0   .007823   .000000   .000005   .000067
 131.0   .009162   .000000   .000005   .000071
 132.0   .010651   .000000   .000005   .000074
 133.0   .012295   .000000   .000005   .000078
 134.0   .014097   .000000   .000006   .000082
 135.0   .016059   .000000   .000006   .000085
-------------------------------------------------

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 160    A = 9
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 135.0   .003607   .000523   .003993   .018114
 136.0   .004340   .000546   .004145   .018[illegible]
 137.0   .005179   .000569   .004297   .019267
 138.0   .006135   .000592   .004449   .019840
 139.0   .007213   .000615   .004602   .020410
 140.0   .008420   .000638   .004754   .020975
 141.0   .009762   .000661   .004905   .021532
 142.0   .011243   .000684   .005054   .022082
 143.0   .012866   .000707   .005202   .022622
 144.0   .014634   .000729   .005347   .023152
 145.0   .016547   .000752   .005490   .023670

-------------------------------------------------
BLOCKING PROBABILITIES FOR C = 160    A = 10
-------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
-------------------------------------------------
 135.0   .003607   .000054   .000621   .004123
 136.0   .004340   .000056   .000649   .004289
 137.0   .005179   .000059   .000678   .004456
 138.0   .006135   .000062   .000707   .004624
 139.0   .007213   .000065   .000736   .004792
 140.0   .008420   .000068   .000766   .004959
 141.0   .009762   .000071   .000795   .005126
 142.0   .011243   .000073   .000824   .005291
 143.0   .012866   .000076   .000853   .005455
 144.0   .014634   .000079   .000882   .005616
 145.0   .016547   .000082   .000910   .005774

-------------------------------------------------
```

```
BLOCKING PROBABILITIES FOR C = 160   A = 11
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
 135.0   .003607   .000004   .000075   .000723
 136.0   .004340   .000005   .000079   .000758
 137.0   .005179   .000005   .000083   .000793
 138.0   .006135   .000005   .000087   .000829
 139.0   .007213   .000005   .000091   .000865
 140.0   .008420   .000006   .000096   .000901
 141.0   .009762   .000006   .000100   .000937
 142.0   .011243   .000006   .000104   .000974
 143.0   .012866   .000006   .000108   .001009
 144.0   .014634   .000007   .000113   .001045
 145.0   .016547   .000007   .000117   .001080

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 160   A = 12
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
 135.0   .003607   .000000   .000007   .000100
 136.0   .004340   .000000   .000008   .000106
 137.0   .005179   .000000   .000008   .000112
 138.0   .006135   .000000   .000009   .000117
 139.0   .007213   .000000   .000009   .000123
 140.0   .008420   .000000   .000010   .000129
 141.0   .009762   .000000   .000010   .000135
 142.0   .011243   .000000   .000011   .000141
 143.0   .012866   .000000   .000011   .000147
 144.0   .014634   .000000   .000011   .000153
 145.0   .016547   .000000   .000012   .000159

--------------------------------------------------
BLOCKING PROBABILITIES FOR C = 160   A = 13
--------------------------------------------------
LOAD       OUTER  INNER(28)  INNER(35)  INNER(42)
--------------------------------------------------
 135.0   .003607   .000000   .000001   .000011
 136.0   .004340   .000000   .000001   .000012
 137.0   .005179   .000000   .000001   .000013
 138.0   .006135   .000000   .000001   .000013
 139.0   .007213   .000000   .000001   .000014
 140.0   .008420   .000000   .000001   .000015
 141.0   .009762   .000000   .000001   .000016
 142.0   .011243   .000000   .000001   .000017
 143.0   .012866   .000000   .000001   .000017
 144.0   .014634   .000000   .000001   .000018
 145.0   .016547   .000000   .000001   .000019
```

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